De Liu, Xianjun Geng, \& Andrew B. Whinston

# Optimal Design of Consumer Contests 


#### Abstract

A consumer contest is a sales promotion technique that requires participants to apply certain skills as they compete for prizes or awards. This article is the first to employ a game-theoretical approach to investigate consumer contest design issues, including prize structure, segmentation, and handicapping. First, the authors find that both skill distribution and the number of contestants play an important role in determining the optimal prize structure in consumer contests. Specifically, if the skill distribution has the increasing hazard-rate property, it is optimal for a marketer to use a winner-take-all design. In large contests, for the winner-take-all approach to be optimal, it suffices to have the increasing hazard-rate property only at the high end of the skill distribution. Second, increasing contest size is beneficial to the marketer. Third, a less dispersive skill distribution leads to higher consumption by consumers at all skill levels and thus is beneficial to the marketer. The marketer may achieve less dispersive skill distributions by (1) segmenting or screening contestants according to their skill levels and (2) adopting a performance evaluation scheme that handicaps high-skilled contestants.


Aconsumer contest is a sales promotion technique that requires participants to apply certain skills as they compete for prizes or awards. For example, several wireless carriers in Europe and Japan hold contests among players in a cell phone-based trivia game called Mobile Millionaire, which resembles the television show "Who Wants To Be a Millionaire?" except it is played with shortmessage service (SMS) or wireless application protocol (WAP). In Mobile Millionaire, players who obtain the highest scores using the shortest time during a given period (e.g., a month) are declared winners and, in most cases, are awarded with prizes up to tens of thousands of dollars. Sports nutrition specialist EAS organizes Body-for-LIFE Challenge, an annual fitness contest with $\$ 1,000,000$ cash prizes that are awarded on the basis of people's before-andafter photographs and their essays about their "inner transformation" during the 12 -week contest period. Two characteristics set consumer contests apart from other sales promotion techniques. First, consumer contests are skill based; that is, skill must play a significant role in determin-

[^0]To read and contribute to reader and author dialogue on JM , visit http://www.marketingpower.com/jmblog.
ing performance. In the first example, skill is the contestants' knowledge, and in the second example, it is the contestants' athleticism. Thus, "lotterylike" contests in which winning is predominately determined by chance ${ }^{1}$ and chance-based prize promotions, including games of chance, lotteries, and sweepstakes, are not consumer contests. ${ }^{2}$ Second, consumer contests reward consumers on the basis of their relative performance. Thus, programs that reward consumers on the basis of absolute criteria, such as frequentflier programs and price discounts, are not consumer contests.

Marketers use consumer contests for various reasons, including increasing overall consumption, encouraging trials, and gaining publicity. This article mainly focuses on the first objective-that is, increasing overall consumption of products and services. Consumer contests are common among marketers that provide products and services that enhance contestants' performance. For example, in Mobile Millionaire, consumers gain higher scores by playing more games, which leads to increased consumption of textmessage service (in the SMS version) or wireless data service (in the WAP version). ${ }^{3}$ In the Body-for-LIFE Challenge example, EAS's sports nutrition products, including Myoplex, Body-for-LIFE, and AdvantEDGE, help consumers achieve their fitness goals. ${ }^{4}$ Such consumer contests are extremely popular in the high-growth online and wire-

[^1]less entertainment industry (Liu, Geng, and Whinston 2007). ${ }^{5}$ Informa Telecoms and Media (2005) estimates that the total revenue of the wireless entertainment industry exceeded $\$ 15$ billion in 2005 and will reach $\$ 42$ billion by 2010. Consumer contests are viewed as one of the most effective means of inducing usage in the wireless entertainment industry (Travish and Smorodinsky 2002). Consumer contests are also increasingly used by Web sites to increase users' staying time and/or number of visits (Hamman 2000), which often lead to more sales or more advertising revenue.

Several design issues arise in a consumer contest. The first set of issues, which we call the "prize structure," involves how many prizes should be offered and how the total award should be allocated among them. Various prize structures can be observed in business practice. For example, carriers of Mobile Millionaire use various prize structures (Table 1), whereas Body-for-LIFE Challenge uses a winner-take-all design. The second set of issues, which we call "contest structure," involves whether and how to segment the consumer population, how to choose a performance evaluation criteria, and whether to handicap highskilled consumers. Marketers use different contest structures. For example, carriers in Japan have designed four stages-normal, bronze, silver, and gold-so that players compete against similarly skilled people in the same stage. Conversely, European carriers run either one contest for all players or two separate contests for SMS and WAP platforms. These stylized observations raise the question of how marketers should optimally design a consumer contest to induce maximal aggregate consumption from contest participants. The objective of this article is to provide marketers with guidelines on contest design issues regarding prize and contest structures.

In light of the preceding discussion, we study a oneperiod model of a consumer contest in which consumers' performance is a multiplicative function of their skill and consumption, and winners are determined by rank order of performance. The central issue for the monopolistic marketer is to choose an optimal design for the contest to maximize the participants' collective consumption. Under this framework, we derive several implications for prize and contest structures.

First, we find that skill distribution and the number of contestants play an important role in determining the optimal prize structure in consumer contests. If the skill distribution has the increasing hazard-rate (IHR) property, it is optimal for the marketer to use a winner-take-all design. Intuitively, the IHR property ensures that competitiveness increases in skill level; thus, high-skilled contestants will compete harder and consume more for the same expected prize than low-skilled contestants. As a result, the winner-take-all design, which is in favor of high-skilled contestants, is optimal. Moreover, in large contests, for the winner-takeall design to be optimal, it suffices to have the IHR property

[^2]TABLE 1
Prize Structures Used in Mobile Millionaire

| Country | Carrier | Prizes |
| :---: | :---: | :---: |
| Czech Republic | Eurotel | -First: \$26,511 <br> -Second: a voucher for \$1,325 for a vacation of winner's choice |
| Ireland | Eircell | -First-tenth: $£ 1,000, £ 500, £ 250$, £125, £64, £32, £20, £20, £20, £20 (in Ireland pounds) |
| United Kingdom | Vodafone | -First: A millionaire weekend for two in Monte Carlo; prize worth £10,000 (US\$14,736), £500 (US\$736) shopping money, $£ 500$ gaming chips to play in the Monte Carlo casino, and dinner with a VIP ticket to a nightclub show <br> -Second-tenth: A bottle of Moet et Chandon Champagne and the WWTBAM board game |
| United Kingdom | Orange | -First: prizes differ every month but do not exceed £2,000 (US\$2,497) |
| Australia | Telstra | -First: AU\$10,000 (US\$4,873) <br> -Second-tenth: AU\$1,000 <br> (US\$487) |
| Belgium | Mobistar | -First: A millionaire weekend for two people worth approximately BE200,000 francs (US\$4,500), including a helicopter flight, use of a sports car, and luxury hotel |
| Chile | Entel PCS | -First-tenth: $\$ 13,900, \$ 6,900$, \$3,500, \$1,700, \$830, \$415, \$280, \$210, \$140, and \$70 |

Notes: Prize data are based on the information on mobile operators' Web sites and marketing materials.
only on the high end of the skill distribution. This is because in large contests, low-skilled contestants' winning chances are too low to be consequential in the marketer's total profit. We also find that if a marketer's objective is to maximize total participation, the resultant optimal prize structure is different; in this case, offering many small prizes is usually better than offering a single prize (as in the winner-take-all design).

Second, increasing contest size is beneficial to the marketer. Increasing the number of contestants has two opposite effects. First, more contestants may consume more; second, each contestant may consume less because of the existence of more rivals. We show that the first effect dominates the second one.

Third, to our knowledge, this is the first study to find that the equilibrium consumption is negatively correlated with the dispersiveness of the skill distribution. In particular, as the skill distribution becomes less dispersive (the gap in skill levels between any two quantiles becomes smaller), the equilibrium consumption increases across all skill lev-
els. Both the dispersiveness and the IHR condition are related to competitiveness, but they are from different angles. That is, a decrease in dispersiveness ensures that competitiveness increases across all skill levels, whereas the IHR condition ensures that competitiveness increases from low to high skill.

The dispersiveness result, together with the effect of contest size on marketer's profits, has many practical implications for contest design. A marketer may achieve a less dispersive skill distribution through screening or segmenting contestants. For example, in some markets, players in the Mobile Millionaire game are divided into gold, silver, and bronze groups according to their demonstrated skills. Alternatively, a marketer can also achieve the effect of a less dispersive skill distribution by increasing the marginal rate of substitution of consumption for skill. Doing so essentially "handicaps" the high-skilled consumers. For example, in the Body-for-LIFE Challenge, a bodybuilder's performance is not evaluated by his or her final body fitness but rather by the difference between his or her body fitness before and after the contest. Thus, the challenge reduces the advantage of more athletic contestants.

We organize the rest of the article as follows: We review related literature and then set forth our model. Our model analysis proceeds in two parts: First, we derive consumers’ equilibrium and analyze the optimal prize structure, and second, we examine contest structure. We then discuss the managerial implications, the limitations of our research, and possibilities for further research.

## Related Literature

A consumer contest is one kind of consumer promotion that product manufacturers and service providers use to target consumers directly (Neslin 2002). Previously addressed consumer promotions include coupons (Gerstner and Hess 1995; Reibstein and Traver 1982), rebates (Soman 1998), and reward programs (Kopalle and Neslin 2003). The consumer contest is distinct from these promotions because it provides incentives on a relative rather than an absolute basis. In addition to providing utilitarian benefits, such as prizes, consumer contests may provide hedonic benefits, such as entertainment value (Chandon, Wansink, and Laurent 2000). Consumer contests are considered particularly useful in generating deep consumer involvement (Feinman, Blashek, McCabe 1986; Kotler 1999).

This article is one of the first to study design issues in consumer contests. To the best of our knowledge, Ward and Hill (1991) provide the only other attempt to address consumer contest design issues. Unlike our game-theoretical approach, however, Ward and Hill draw on cognitive and social psychology to recommend contest designs.

Our research on consumer contests also contributes to the broader literature on contests, in which game-theoretical modeling is a major analytical tool. These include rankorder tournaments (Lazear and Rosen 1981; Nalebuff and Stiglitz 1983), sales contests (Basu et al. 1985; Kalra and Shi 2001), rent seeking (Tullock 1981), lobbying (Baye, Kovenock, and De Vries 1993), research-and-development (R\&D) tournaments (Che and Gale 2003; Fullerton and

McAfee 1999; Taylor 1995), and patent races (Dasgupta and Stiglitz 1980). Baye and Hoppe (2003) show a strategic equivalence among rent seeking, patent races, and $R \& D$ tournaments. Consumer contests differ from other types of contests in design objectives. In most other contests, designers take direct interests in contestants' performance. For example, the designers of sales contests are interested in the aggregate performance of all salespeople (Kalra and Shi 2001) and the designers of R\&D tournaments are interested in the highest performance (Fullerton and McAfee 1999). However, designers of consumer contests do not benefit directly from consumers' performance per se; rather, they make their profits from induced consumption-text messages in Mobile Millionaire and nutritional products in Body-for-LIFE Challenge. As such, research on other contest types does not address the design problems in consumer contests.

We adopt an all-pay auction framework for studying consumer contests. The all-pay auction, in which bidders pay what they bid regardless of winning, is widely used to model various contest settings, including lobbying (Baye, Kovenock, and De Vries 1993), patent races (Dasgupta and Stiglitz 1980), and R\&D contests (Moldovanu and Sela 2001). All-pay auctions have been applied in other contestlike settings, such as Varian's (1980) model of sales and Iyer and Pazgal's (2003) model of Internet shopping agents. Our model framework is similar to the one used by Moldovanu and Sela (2001) because both assume that contestants have different skill levels.

Several studies have examined the design of prize structures in tournaments or sales contests. Krishna and Morgan (1998) find that the winner-take-all design is optimal when the number of competing workers is two or three. Kalra and Shi (2001) study a multiple-player model of sales contests and find that the number of prizes should be increased and the spread should be decreased when salespeople are more risk averse. Both studies assume that contestants have identical skills and that performance is stochastic. In contrast, in our model, contestants differ in their skills, and performance is deterministic. In reality, however, a contest often involves both heterogeneously skilled participants and stochastic outcomes, which leads us to believe that these studies and our own examine two extremes of the general case and thus complement each other.

Glazer and Hassin (1988) and Moldovanu and Sela (2001) study the design of prize structures in contests in which contestants differ in skills. Glazer and Hassin show that the winner-take-all design is optimal if the skill distribution is uniform and the performance function is linear. Moldovanu and Sela show that the winner-take-all design is optimal under general distributions, as long as the performance function is linear. Both studies assume that the designer maximizes aggregate performance-the variable used to rank contestants. In contrast, our study assumes that the designer maximizes aggregate consumption, which is not directly used for ranking and serves as an input factor of performance. Because of the distinction in design objectives, skill distribution and the number of contestants play critical roles in our study, but this is not the case in the other studies. In general, their results for the winner-take-all
design do not hold in our setting．Moreover，our discussion on the dispersiveness of the skill distribution and its many implications for the designer＇s profits is entirely novel．

A few authors have studied handicapping in contest set－ tings．Lazear and Rosen（1981）show that，conditional on knowing workers＇type，awarding a handicap to the inferior worker can induce efficient investment from both workers． Feess and Muehlheusser（2003）and Clark and Riis（1998） study handicapping in auction settings in which the bid from one bidder is weighted differently from that of another．In all these studies，handicapping requires knowl－ edge of contestants＇identities．In our case，however，handi－ capping is not based on contestants＇identities；rather，it is but embedded in the performance function of the contest．In our article，handicapping also takes a more general form．

## A Game－Theoretical Model of Consumer Contests

We consider that a monopoly marketer sells one type of goods（product or service），for which it charges a constant unit price p ．There are a total of n consumers for the goods， indexed by $\mathrm{i}=1,2, \ldots, \mathrm{n}$（we drop the subscription i when－ ever we talk about any one consumer）．

To promote the goods，the marketer organizes a con－ sumer contest among the n customers．A consumer＇s perfor－ mance in the consumer contest，$x$ ，is affected by both the amount of goods he or she consumes（hereinafter，＂con－ sumption＂），$t$ ，and parameter $\mu$ ，which we interpret as his or her skill．For example，in Mobile Millionaire，players＇high scores（performance）are determined by both their knowl－ edge（skill）and the number of games they play，which in turn determines the number of text messages or WAP min－ utes they consume（consumption）．

Depending on the application setting，the consumption factor，$t$ ，can be interpreted either as new consumption， when the market for the goods is new and the consumer contest is designed to induce trial，or as incremental con－ sumption due to the contest，when there is already a market for the goods and a consumer contest is designed to induce incremental consumption．We assume that a consumer＇s marginal intrinsic utility $\theta$ from consumption（under the first interpretation）or from incremental consumption （under the second interpretation）is less than p．Consumers＇ valuations for new products or services are below prices because of uncertainty and a lack of full appreciation．Con－ sumers＇valuations for incremental consumption are below prevailing market price because of decreasing marginal util－ ity．In what follows，we no longer differentiate between the two interpretations but refer to $t$ simply as＂consumption．＂

We interpret the skill factor as an endowed characteris－ tic of a consumer at the beginning of the contest．${ }^{6}$ Each con－ sumer＇s skill factor is drawn independently from a common

[^3]distribution $F(\mu)$ with a finite support $[\underline{\mu}, \bar{\mu}]$ ，where $\bar{\mu}>\underline{\mu}>0$ ．Its density function， $\mathrm{f}(\mu)$ ，is continuous and posi－ tive．We assume that consumers know their own skill fac－ tors but that other consumers and the marketer do not．Both $F(\mu)$ and $n$ are common knowledge．

We assume that a consumer＇s performance depends on consumption and skill according to a multiplicative perfor－ mance function，${ }^{7} \mathrm{x}(\mu, \mathrm{t})=\mu \mathrm{t}$ ．This performance function increases in both skill and consumption；furthermore，the skill factor and the consumption factor reinforce each other， in the sense that the marginal effect of consumption（skill） on performance increases in skill（consumption）．

The marketer gives prizes according to the rank order of consumers＇performance．Let $\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{n}}$ denote sizes of the prizes，$v_{1} \geq v_{2} \geq \ldots \geq v_{n} \geq 0$ ．We call（ $v_{1}, v_{2}, \ldots, v_{n}$ ）a prize structure．We index the prizes by j and denote the prize sum as $\mathrm{V}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{v}_{\mathrm{j}}$ ．

We define the marketer＇s spending on the jth prize dif－ ferential as

$$
\begin{equation*}
\delta_{\mathrm{j}} \equiv \mathrm{j}\left(\mathrm{v}_{\mathrm{j}}-\mathrm{v}_{\mathrm{j}+1}\right), \mathrm{j}=1, \ldots, \mathrm{n}\left(\text { let } \mathrm{v}_{\mathrm{n}+1} \equiv 0\right) \tag{1}
\end{equation*}
$$

A prize structure can be equivalently represented by a set of prize differentials．In particular，the sum of prize differen－ tials equals the sum of prizes and

$$
\begin{equation*}
\mathrm{v}_{\mathrm{j}}=\sum_{l=\mathrm{j}}^{\mathrm{n}} \delta_{l} / l, \mathrm{j}=1, \ldots, \mathrm{n} \tag{2}
\end{equation*}
$$

Figure 1 illustrates that a prize structure（．4，．2，．2，．1，．1，0） （the vertical bars）can be represented by prize differentials
${ }^{7}$ Under the first interpretation，the performance is a function of the skill factor and the overall new consumption．Under the second interpretation，the performance is a function of the skill factor under the base consumption（without the contest）and the incre－ mental consumption．For example，a consumer who plays Mobile Millionaire a lot even without a contest may have an advantage in the contest because of improved skill through practice．Note also that our results will not change if we adopt a seemingly more gen－ eral functional form $x(\mu, t)=g(\phi(\mu), t)$ ，where $g^{\prime}>0, g(0)=0$ ，and $\phi^{\prime}>0$ ．This is because only relative performance matters，and we can redefine $\phi(\mu)$ as skill．The essential assumption is the multi－ plicative relationship between consumption and skill．

FIGURE 1
Prize Structure and Spending on Prize Differentials

| Prize Size |  |  |  | spending on： |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\square$ First prize differential |  |
|  | First prize |  |  |  |  |
|  |  |  |  | QThird prize differential |  |
| ． 4 |  |  |  | Fifth prize differential |  |
| ． 3 |  | Second | Third |  |  |
| ． 2 |  | ？ | $\square$ | Fourth | Fifth |
|  |  |  |  | prize | prize |
| ． 1 |  | V\％ | $\ddot{\square}$ | W／b | W／ |
|  |  | 厉 | 年品 | W10 | W析 |

$(.2,0, .3,0, .5,0)$ (the horizontal strips). In this article, we often work with prize differentials because the order constraint $\mathrm{v}_{1} \geq \mathrm{v}_{2} \geq \ldots \geq \mathrm{v}_{\mathrm{n}}$ on prize structure can be conveniently represented by $\delta_{\mathrm{j}} \geq 0, \mathrm{j}=1, \ldots, \mathrm{n}$ using prize differentials.

## A Consumer's Utility Function

Consumers maximize their total expected utility, which we assume is additive in prizes, intrinsic utility from consumption, and payment. The total utility ( U ) of a consumer who participates in the contest with performance x and skill level $\mu$ is
(3) $U=\sum_{j=1}^{n} v_{j} \operatorname{Pr}\{x$ is the jth highest performance $\}-(p-\theta) \frac{x}{\mu}$

$$
\begin{aligned}
& =\sum_{j=1}^{n}\left(v_{j}-v_{j+1}\right) \operatorname{Pr}\{\mathrm{x} \text { is at least the jth highest }\}-(\mathrm{p}-\theta) \frac{\mathrm{x}}{\mu} \\
& =\sum_{\mathrm{j}=1}^{\mathrm{n}} \frac{1}{\mathrm{j}} \delta_{\mathrm{j}} \operatorname{Pr}\{\mathrm{x} \text { is at least the jth highest }\}-(\mathrm{p}-\theta) \frac{\mathrm{x}}{\mu} .
\end{aligned}
$$

## The Marketer's Profit Function

The marketer's contest revenue comes from the consumption stimulated by the contest. We assume that the marketer's unit cost of providing the goods is a constant, normalized to zero. The cost of running the contest is a function of the prize sum $(\mathrm{V})$ offered, $\mathrm{c}(\mathrm{V})$. The cost of running the contest may include the cost of providing the prizes, hiring lawyers to write legal documents, handling registration or performance evaluation, advertising for the contest, and so on. We assume that $\mathrm{c}(0)=0, \mathrm{c}^{\prime}(\cdot)>0$, and $c^{\prime \prime}(\cdot)>0$, where $c(0)=0$ implies that if the marketer does not run a consumer contest, it will incur no cost, and $c^{\prime \prime}(\cdot)>$ 0 implies that the marginal cost of running the contest increases in the prize sum. The latter is because, for example, contests with a larger prize sum are subject to more legal scrutiny. We assume that the marketer is risk neutral. The expected total profit of the marketer is

$$
\begin{equation*}
\pi=n p E[t]-\mathrm{c}\left(\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}}\right) \tag{4}
\end{equation*}
$$

where $\mathrm{E}[\mathrm{t}]$ is the expected consumption of a consumer in the contest.

## The Consumer Contest

The consumer contest unfolds in the following way: The marketer announces the prize structure and the contest rules before the contest starts. Meanwhile, consumers learn their private skill factor $\mu$, the common distribution F , and the number of consumers $n$. During the contest, each consumer simultaneously and independently chooses his or her consumption t . After the contest ends, the marketer evaluates each consumer's performance and awards prizes according to the announced rules.

## The Optimal Prize Structure

## Equilibrium Consumption and Profit

A consumer's strategy is a mapping from the consumer's skill factor to consumption level. A strategy profile for all consumers constitutes a Nash equilibrium if no one has the incentive to deviate from his or her own strategy, provided that others do not. We consider only the symmetric-strategy Nash equilibrium, in the sense that all consumers adopt the same strategy function, denoted as $t(\mu)$, in equilibrium. Let $\mathrm{x}(\mu)=\mathrm{t}(\mu) \mu$ denote the corresponding equilibrium performance.

If the equilibrium performance $\mathrm{x}(\mu)$ is monotonically increasing (in the proof of $\mathrm{P}_{1}$, we verify that it is), then
(5) $\operatorname{Pr}\{\mathrm{x}(\mu)$ is at least the jth highest $\}=\operatorname{Pr}\{\mu$ is at least the

$$
\begin{aligned}
& \text { jth highest }\} \\
& =\operatorname{Pr}\left\{\mu_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1} \leq \mu\right\},
\end{aligned}
$$

where $\mu_{n-j: n-1}$ is the $(n-j)$ th lowest (or jth highest) order statistics from $n-1$ samples. The second step in Equation 5 is because the probability of $\mu$ being at least the jth highest in $n$ samples is the same as the probability that the jth highest in $n-1$ samples is less than $\mu$. Let $\mathrm{F}_{\mathrm{j}: \mathrm{n}}$ and $\mathrm{f}_{\mathrm{j}: \mathrm{n}}$ denote the cumulative distribution function and the probability density functions for $\mu_{j: n}$, respectively.

The formula for $\mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu)$ and $\mathrm{f}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu)$ are, respectively, ${ }^{8}$
(6) $\mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu)=\sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1}$,

$$
\text { for } \mathrm{j}=1, \ldots, \mathrm{n}-1 \text {, and }
$$

$$
\begin{gather*}
f_{n-j: n-1}(\mu)=j\binom{n-1}{n-j-1} F(\mu)^{n-j-1[1-F(\mu)]^{j}-1 f(\mu)}  \tag{7}\\
f \text { for } j=1, \ldots, n-1
\end{gather*}
$$

$P_{1}$ (equilibrium consumption): There is a unique symmetricstrategy Nash equilibrium, in which a consumer's strategy is given by

$$
\begin{equation*}
\mathrm{t}(\mu)=\frac{1}{\mathrm{p}-\theta} \sum_{\mathrm{j}=1}^{\mathrm{n}}\left[\delta_{\mathrm{j}} \frac{1}{\mu} \int_{\underline{\mu}}^{\mu} \sigma_{-1}^{1} \mathrm{f}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma) \mathrm{d} \sigma\right] . \tag{8}
\end{equation*}
$$

(For the proof, see the Appendix.)
The consumer's equilibrium consumption is a weighted sum of the marketer's spending on all prize differentials. Consumers with different skill levels assign different weights to prize differentials. The equilibrium consumption decreases with disutility of incremental consumption ( $\mathrm{p}-$ $\theta$ ). Note that the equilibrium consumption may not always increase with skill level, though the equilibrium performance does (see the Appendix).

Substituting Equation 8 into Equation 4, we can obtain the marketer's expected profit as

[^4]\[

$$
\begin{gather*}
\pi=\sum_{j=1}^{n}\left[\delta_{j} \frac{n p}{p-\theta} \int_{\underline{\mu}}^{\bar{\mu}} \frac{1}{\mu} \int_{\underline{\mu}}^{\mu} \sigma-{\underset{j}{j}}_{1} f_{\mathrm{j}-\mathrm{n}-1}(\sigma) \mathrm{d} \sigma f(\mu) \mathrm{d} \mu\right]  \tag{9}\\
-\mathrm{c}\left(\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{j}\right)
\end{gather*}
$$
\]

We also denote

$$
\begin{equation*}
\varepsilon_{j} \equiv \mathrm{n} \frac{\mathrm{p}}{\mathrm{p}-\theta} \int_{\underline{\mu}}^{\bar{\mu}} \frac{1}{\mu} \int_{\underline{\mu}}^{\mu} \sigma \frac{1}{j_{j}} \mathrm{f}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma) \operatorname{d} \sigma f(\mu) \mathrm{d} \mu, \tag{10}
\end{equation*}
$$

where $\varepsilon_{\mathrm{j}}$ represents the revenue generated by unit spending on the jth prize differential. We can rewrite Equation 9 as

$$
\begin{equation*}
\pi=\sum_{j=1}^{n} \delta_{\mathrm{j}} \varepsilon_{\mathrm{j}}-\mathrm{c}\left(\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}}\right) \tag{11}
\end{equation*}
$$

Similar to the equilibrium consumption, the expected revenue $\left(\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}} \varepsilon_{\mathrm{j}}\right)$ is a weighted sum of spending on all prize differentials.

## A General Result on the Optimal Prize Structure

The optimal prize structure problem for the marketer is to maximize its total profits (Equation 11) by choosing its spending on different prize differentials $\left(\delta_{\mathrm{j}}\right)$. The partial derivative of Equation 11 is $\partial \pi / \partial \delta_{j}=\varepsilon_{j}-c^{\prime}\left(\sum_{j=1}^{n} \delta_{j}\right)$ for $j-1, \ldots, n$. Therefore, for $j, k=1, \ldots, n$ and $j \neq k, \partial \pi / \partial \delta_{j}-$ $\partial \pi / \partial \delta_{\mathrm{k}}=\varepsilon_{\mathrm{j}}-\varepsilon_{\mathrm{k}}$, which does not depend on $\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}}$. In other words, the optimal allocation of spending among prize differentials does not depend on the total prize sum.

This implies that we can decompose the original problem into two subproblems: (1) a prize allocation problem (i.e., to determine the optimal prize allocation for an arbitrary prize sum) and (2) a prize budget problem (i.e., to determine the optimal prize sum given the optimal prize allocation derived in Subproblem 1).

## $\mathrm{P}_{2}$ (optimal prize structure):

(a) (optimal prize allocation): It is optimal to allocate the entire prize sum to the most profitable prize differential, $j^{*} \equiv \arg \max _{\mathrm{j}}\left\{\varepsilon_{\mathrm{j}}\right\}$.
(b) (optimal prize sum): Let $\mathrm{j}^{*}$ be the most profitable prize differential. If $\mathrm{c}^{\prime}(0) \geq \varepsilon_{j^{*}}$, the marketer should not run a consumer contest; if $\mathrm{c}^{\prime}(0)<\varepsilon_{j^{*}}$, the optimal prize sum $\mathrm{V}^{*}$ is the solution to

$$
\begin{equation*}
\varepsilon_{j^{*}}=c^{\prime}\left(\mathrm{V}^{*}\right) \tag{12}
\end{equation*}
$$

$P_{2 a}$ prescribes that when the first prize differential is the most profitable $\left(\mathrm{j}^{*}=1\right)$, the marketer should allocate the entire prize sum to the first prize. The resultant prize structure is a winner-take-all structure. When the most profitable prize differential is $\mathrm{j}^{*} \geq 2$, the marketer should split the prize sum equally among the first $\mathrm{j}^{*}$ prizes. The resultant prize structure is a split-prize structure.
$P_{2 b}$ shows that the optimal prize sum is determined by two factors: the marginal revenue generated in the consumer contest under the optimal prize allocation and the marginal cost of running the contest. It is never optimal for
the marketer to award the nth prize. Intuitively, the nth prize differential does not provide any incentive for consumption, because a consumer can obtain it by not consuming at all. Hereinafter, we consider only a maximum of $n-1$ prizes.
$P_{2}$ gives clear guidelines on how to select prize structures optimally. We begin by computing revenues generated by unit spending on different prize differentials (according to Equation 10); then, we compare them to determine the optimal prize allocation. The optimal prize sum can then be determined according to Equation 12. The following examples demonstrate the use of $\mathrm{P}_{2}$. They also show that both the winner-take-all and the split-prize designs can be optimal. In both examples, let $\mathrm{n}=5, \mathrm{p} /(\mathrm{p}-\theta)=2$, and $\mathrm{c}(\mathrm{V})=\mathrm{V}^{2} / 2$.

Example 1 (the winner-take-all design): Skill is uniformly distributed on [1,2]. The values of $\varepsilon_{1}-\varepsilon_{4}$ are 1.820, $1.721,1.617$, and 1.506 , respectively. Thus, the optimal prize allocation is $(\mathrm{V}, 0,0,0,0)$, and the optimal prize sum is $\mathrm{V}=1.820$.
Example 2 (the split-prize design): We consider a doublepeaked skill distribution on $[1,10]$ with a majority being low skilled. The density function is given by ${ }^{9}$

$$
\mathrm{f}(\mu)= \begin{cases}.9, & \mu \in[1,2) \\ 0, & \mu \in[2,9) . \\ .1, & \mu \in[9,10]\end{cases}
$$

The marginal revenues of the first to the fourth prize differential are 1.356, 1.407, 1.411, and 1.357, respectively. Therefore, the optimal prize allocation is $(.33 \mathrm{~V}, .33 \mathrm{~V}, .33 \mathrm{~V}$, 0,0 ), and the optimal prize sum is $\mathrm{V}=1.411$.
$P_{2}$ shows that the optimal prize allocation depends critically on the rank order of $\varepsilon_{\mathrm{j}}$. Therefore, it becomes relevant to ask how the rank order of $\varepsilon_{\mathrm{j}}$ is affected by the underlying factors of the consumer contest. In the next subsection, we study two such factors: skill distribution and the number of contestants.

## The Effect of Skill Distribution and $n$ on Optimal Prize Structure

Let $h(\mu)=f(\mu) /[1-F(\mu)]$ denote the hazard rate of $F$ at $\mu$. In this study, the hazard rate at $\mu$ represents the instantaneous probability that a consumer's skill factor is $\mu$, provided that the consumer's skill factor is no less than $\mu$.
-Definition: A distribution function F is said to be (or to have the property of) IHR if

$$
\begin{equation*}
\mathrm{h}(\mu) \text { is increasing in } \mu \text { on }(\underline{\mu}, \bar{\mu}) . \tag{13}
\end{equation*}
$$

The IHR property implies that the higher a consumer's skill level, the higher is the probability that he or she will face

[^5]opponents of similar skills, compared with the probability that he or she will face opponents of higher skills. The IHR property is satisfied when the density of the distribution is single peaked and does not drop too fast. Examples of IHR distributions include uniform, normal, logistic, exponential, and any distributions with nondecreasing density.
$P_{3}$ : A winner-take-all prize allocation is optimal if the skill distribution has the IHR property.

The intuition of $\mathrm{P}_{3}$ comes from a close examination of the incentive effect of different prize differentials across skill levels. As Figure 2 illustrates, the first prize differential motivates the highest-skilled consumers more than does the second (and any other) prize differential in the sense that it generates higher revenue from them. As we mentioned previously, the IHR property implies that the higher a contestant's skill level, the more likely he or she will face opponents with similar skills than opponents of higher skills. Because facing opponents with similar skills encourages competition (and facing opponents that are too good discourages competition), the IHR property suggests that highskilled consumers compete more aggressively than lowskilled consumers, thus contributing more to the total revenue. Thus, from the marketer's point of view, highskilled consumers are more profitable. Because the first prize differential is the most effective in motivating the high-skilled consumers, the marketer should allocate the entire prize budget to the first prize differential, which implies a winner-take-all prize structure.
$P_{3}$ can be further generalized to the case in which the skill distribution has an increasing (and higher) hazard rate only on the right side of its support.
-Definition (pseudo-IHR): A distribution function F is said to be (or to have the property of) pseudo-IHR if there is a starting point $\chi$ such that

FIGURE 2
Revenue Generated by Unit Spending on the First and Second Prize Differentials


$$
\begin{equation*}
\mathrm{h}(\mu) \leq \mathrm{h}(\chi) \text { for any } \mu \in[\underline{\mu}, \chi] . \tag{14}
\end{equation*}
$$

$\mathrm{P}_{4}$ : If the skill distribution has the pseudo-IHR property with starting point $\chi$, the winner-take-all prize structure is optimal for $\mathrm{n}>\mathrm{n}_{0}$, where $\mathrm{n}_{0}$ satisfies the following:

$$
\begin{equation*}
\left[\frac{1}{\mathrm{~F}(\chi)}\right]^{\mathrm{n}_{0}-1}-\left[\frac{1-\mathrm{F}(\chi)}{\mathrm{F}(\chi)}\right]^{\mathrm{n}_{0}-1}=\mathrm{n}_{0}-1 \tag{15}
\end{equation*}
$$

$\mathrm{P}_{4}$ implies that for a pseudo-IHR distribution, the winner-take-all design will eventually become optimal as contest size increases. The intuition of $\mathrm{P}_{4}$ is as follows: When the size of the consumer contest increases, the probability of winning a prize diminishes more quickly for lowskilled consumers than for high-skilled ones; thus, the consumption level of low-skilled consumers drops faster than that of their high-skilled counterparts (Figure 3). Because the low-skilled segment contributes less to the total revenue, it becomes increasingly insignificant in determining the optimal prize allocation. As a result, when $n$ is large enough, an increasing and higher hazard rate at the highskill level is sufficient for the winner-take-all design to be optimal.

The following example confirms $\mathrm{P}_{4}$ 's prediction:

> Example 3: To continue from Example 2, we can verify that the skill distribution does not satisfy the IHR property but rather satisfies a pseudo-IHR property. Table 2 shows that a split-prize structure is optimal for n up to 16 . Beyond that, the winnertake-all design becomes optimal.

However, we must caution readers not to draw a general conclusion about when a consumer population is large enough without studying the underlining skill distribution. In some cases, it may require a large number of consumers for the winner-take-all design to become optimal. In Example 4 , for n up to 100 , the optimal prize structure is still a split-prize design (and it becomes difficult to compute numerical results beyond 100).

Example 4: We can verify that the following density function satisfies the pseudo-IHR property (but not the IHR property) with the starting point $\chi$ close to the upper bound of the support:

$$
f(\mu)= \begin{cases}.1, & \mu \in[1,2) \\ 0, & \mu \in[2,8.5) \\ 1.78, & \mu \in[8.5,9) \\ .01, & \mu \in[9,10]\end{cases}
$$

When $\mathrm{P}_{3}$ and $\mathrm{P}_{4}$ are combined, they have the following implications for marketers: The skill distribution and the number of consumers determine whether the prize structure should motivate the high-skilled segment. If the density of the skill distribution is single peaked and does not drop too fast (so that IHR is satisfied), the high-skilled segment is profitable enough to ensure that a winner-take-all prize structure (which is most effective in motivating the highskilled consumers) is optimal ( $\mathrm{P}_{3}$ ). Violation of IHR may

FIGURE 3
Effect of $\boldsymbol{n}$ on the Marginal Revenue Generated by the First and Second Prize Differentials


TABLE 2
Effect of Size on Optimal Prize Structure

| $\mathbf{n}$ | Optimal Number <br> of Prizes | Expected <br> Revenue |
| :---: | :---: | :---: |
| 10 | 5 | 1.441 |
| 11 | 6 | 1.442 |
| 12 | 6 | 1.445 |
| 13 | 7 | 1.447 |
| 14 | 8 | 1.449 |
| 15 | 8 | 1.450 |
| 16 | 8 | 1.452 |
| 17 | 1 | 1.459 |
| 18 | 1 | 1.483 |
| 19 | 1 | 1.507 |
| 20 | 1 | 1.530 |

suggest that a split-prize allocation is more effective because of its ability to motivate middle- or low-skilled consumers. For example, when there are "pro" consumers with substantial skill advantage (as in Example 4), it may be optimal for the marketer to award many small prizes to keep middle- or low-skilled consumers motivated. ${ }^{10}$ Moreover, our results suggest that as the size of the contest increases, the choice of prize structure is increasingly determined by the high end of the skill distribution because low-skilled consumers' winning chances and their contribution to the marketer's revenue diminish quickly $\left(\mathrm{P}_{4}\right)$.

The intuition we obtain here is novel compared with that of Moldovanu and Sela (2001). In their analysis, the reason for the contest designer to adopt a non-winner-takeall prize structure is that contestants have convex disutility.

[^6]Conversely, in this article, factors that drive different prize structures are skill distribution and the number of contestants. Our results contribute to the contest design literature by showing that the winner-take-all-is-always-optimal result Moldovanu and Sela obtain in the linear disutility setting is susceptible to a perturbation of the designer's objective function.

## Prize Structure and Contest Participation

We now turn our attention to the effect of prize structure on participation in consumer contests. This issue may be of interests to marketers that care about the reach of their promotion rather than the total profit. We assume that each consumer incurs a fixed entry cost, e, on participating in the consumer contest. Such a cost may be interpreted as the monetary or psychological cost involved in preparing for the contest. For example, in the Body-for-LIFE Challenge, a contestant may need to purchase workout gear and to register for the contest.

Because the entry cost is incurred at the outset, consumers whose expected payoff is not high enough to offset the entry cost will not participate. Because consumers' expected payoff from a contest increases with their skill levels, there is a marginal consumer who is indifferent between participating and not participating. We denote the skill quantile of the marginal consumer as $u^{*}$. In what follows, we use $\mathrm{u}^{*}$ as a measure of participation. We can state the problem of maximizing participation as

$$
\begin{gather*}
\min _{\left\{\delta_{1}, \delta_{2}, \ldots, \delta_{n-1}\right\}} u^{*}  \tag{16}\\
\text { subject to } \sum_{j=1}^{n-1} \delta_{j} P_{j}\left(u^{*}\right)=e \text { and } \sum_{j=1}^{n-1} \delta_{j}=V
\end{gather*}
$$

where $P_{j}(u) \equiv F_{n-j: n-1}\left(F^{-1}(u)\right) / j$ is the probability of winning the jth prize differential. The following proposition
characterizes the minimal cutoff skill quantile $u^{*}$ and the corresponding prize structure:
$\mathrm{P}_{5}$ : Let $\overline{\mathrm{P}}(\mathrm{u}) \equiv \max _{\mathrm{j}}\left\{\mathrm{P}_{\mathrm{j}}(\mathrm{u})\right\}$ be the upper envelope of the function family $\left\{\mathrm{P}_{\mathrm{j}}(\mathrm{u}), \mathrm{j}=1, \ldots, \mathrm{n}-1\right\}$. The minimal cutoff skill quantile $\mathrm{u}^{*}$ (maximum participation) is the solution to

$$
\begin{equation*}
\overline{\mathrm{P}}\left(\mathrm{u}^{*}\right)=\frac{\mathrm{e}}{\mathrm{~V}}, \tag{17}
\end{equation*}
$$

and the corresponding prize structure is to split V among the first $\mathrm{j}_{\mathrm{u}^{*}}$ prizes, where $\mathrm{j}_{\mathrm{u}^{*}}$ is the index of the member function on the envelope at $u^{*}$ and $\mathrm{j}_{\mathrm{u}^{*}}$ (weakly) decreases in $\mathrm{e} / \mathrm{V}$ and is independent of the skill distribution $\mathrm{F}(\mu)$.

The main implication of $\mathrm{P}_{5}$ is that the marketer should use a large prize sum but small prize sizes to attract the broadest participation. To maximize participation, for a given entry cost e and prize sum V , the marketer should choose a prize structure that maximizes expected payoff or the cutoff skill quantile $u^{*}$. We also know that for some $\mathrm{u}_{\mathrm{n}-1}<\ldots<\mathrm{u}_{1}<1$, the first prize differential generates the highest expected payoff for skill quantiles between $\mathrm{u}_{1}$ and 1, the second prize differential generates the highest expected payoff between $\mathrm{u}_{2}$ and $\mathrm{u}_{1}$, and so on. Because $\overline{\mathrm{P}}(\mathrm{u})$ increases in $u$, it is clear that as e/V decreases, $u^{*}$ decreases so that more prizes should be given to maximize participation. The prize allocation is not affected by the shape of the skill distribution because what matters in a participationmaximization problem is the winning probability of a particular skill quantile, which is the same across all distributions (as long as a high-skilled consumer wins over a low-skilled one).

## Contest Structure

In this section, we study several other design decision variables. We begin by analyzing the impact of consumer population size and skill distribution on participants' consumption and marketer profits. We then discuss their implications on several contest design issues, including segmenting contestant population and designing performance functions.

## The Impact of Size on Profits

$\mathrm{P}_{6}$ (size and profit): When the condition in $\mathrm{P}_{3}$ or the conditions in $\mathrm{P}_{4}$ are satisfied, the marketer's profit increases in n .

Whether the total profit will increase depends on the trade-off between two opposite effects: New participants directly contribute to the aggregate consumption, but they also indirectly cause incumbents to lower their consumption levels because each incumbent has a lower (marginal) chance of winning. $\mathrm{P}_{6}$ shows that the conditions in $\mathrm{P}_{3}$ or $\mathrm{P}_{4}$ guarantee that the direct effect dominates the indirect effect. Intuitively, when the number of consumers increases, the competitiveness increases and the probability of winning a prize diminishes more quickly for lower-skilled consumers. As a result, prizes are more likely to go to higher-skilled consumers. Such a reallocation of winning probabilities increases total revenue because the IHR condition (or the pseudo-IHR condition) guarantees that higher-skilled consumers generate higher revenue for the same prize. Figure 4 illustrates the profit trend when skill is uniformly distrib-

uted on $[1,2], \mathrm{p} /(\mathrm{p}-\theta)=2$, and the prize structure is the winner-take-all design.

## The Impact of Skill Distribution on Profits

There are many reasons for studying the impact of skill distribution on equilibrium consumption and profits. For example, a marketer can influence the skill distribution of a consumer contest through admission policy and segmentation (we discuss this subsequently). To the best of our knowledge, no prior research has directly addressed the issue of how skill distribution affects the marketer's profits, despite its obvious relevance to practice.

To study the impact of skill distribution, it is necessary to compare one distribution with another. The tool we use is called "dispersive order," which we define as follows:
-Definition (dispersive order): Let $\mathrm{F}^{-1}(\mathrm{u})$ and $\mathrm{G}^{-1}(\mathrm{u}), \mathrm{u} \in$ $[0,1]$, be the inverse of $F$ and $G$, respectively. If

$$
\begin{equation*}
\frac{\mathrm{F}^{-1}(\mathrm{u})}{\mathrm{F}^{-1}(\mathrm{~s})} \leq \frac{\mathrm{G}^{-1}(\mathrm{u})}{\mathrm{G}^{-1}(\mathrm{~s})}, \text { whenever } 0<\mathrm{s} \leq \mathrm{u}<1, \tag{18}
\end{equation*}
$$

then F is said to be less dispersive than G. ${ }^{11}$
Intuitively, a distribution is less dispersive when the gap between any two skill quantiles (e.g., $50 \%$ and $60 \%$ skill levels) is smaller.
$\mathrm{P}_{7}$ (skill distribution and profit): The equilibrium consumption systematically decreases as the skill distribution becomes more dispersive.

We provide a formal representation of this proposition in the Appendix. $\mathrm{P}_{7}$ implies that a marketer can expect a higher consumption level in every skill level if the skill distribution is less dispersive. The linkage between equilibrium consumption and skill dispersiveness is due to the follow-

[^7]ing: When the skill distribution is less dispersive, it is easier, for example, for a $50 \%$ skill level to beat a $60 \%$ skill level, which implies that both consumers will compete more aggressively (and consume more). The same logic applies to consumers at all skill levels; thus, the equilibrium consumption is higher across all skill levels. The intuition of this proposition is consistent with our everyday observation: Competition is fiercer in closely matched contests.

Note that though both the dispersiveness measure and the IHR condition affect the competitiveness of a consumer contest, they work from different angles. That is, a decrease in dispersiveness ensures that competitiveness increases across all skill levels, whereas the IHR condition ensures that competitiveness increases from low to high skill.
$\mathrm{P}_{7}$ has many implications for designing consumer contests. For example, a marketer can achieve a less dispersive skill distribution by excluding certain low-skilled (or highskilled) consumers, by segmenting consumers into different skill groups (which we discuss in the next subsection), or by assisting low-skilled consumers to gain skill levels. Another set of implications is related to designing the performance function. The following corollary shows that the marketer can achieve the same effect of a less dispersive distribution by altering a performance function in a certain way:

> Corollary 1 (design of performance function): Consider two performance functions: $X^{A}(\mu, t)=\mu t$, and $X^{B}(\mu$, $t)=\phi(\mu) t$, where $\phi(\cdot)$ is an increasing function. All else being equal, $X^{B}(\mu, t)$ generates systematically higher equilibrium consumption than $\mathrm{X}^{\mathrm{A}}(\mu, \mathrm{t})$ if

$$
\begin{equation*}
\frac{\phi(\mu)}{\phi(\sigma)}<\frac{\mu}{\sigma}, \text { for any } \underline{\mu} \leq \sigma<\mu \leq \bar{\mu} . \tag{19}
\end{equation*}
$$

To understand Corollary 1, we can redefine $\mu^{\prime} \equiv \phi(\mu)$ as skill. Under Equation 19, $\mu^{\prime}$ has a less dispersive distribution than $\mu$ and thus permits systematically higher equilibrium consumption $\left(\mathrm{P}_{7}\right)$. We also show that the condition in Equation 19 is equivalent to requiring that $\mathrm{X}^{\mathrm{B}}(\mu, \mathrm{t})$ has a higher marginal rate of substitution of consumption for skill than $\mathrm{X}^{\mathrm{A}}(\mu, \mathrm{t})$ (see the Appendix). The intuition is clearer from the latter view: If the marketer can choose a performance function in which consumption is a stronger substitute for skill, it essentially "handicaps" high-skilled consumers (because they suffer the most from a lesser role of the skill factor). Such a handicapping approach enhances competitiveness and stimulates a higher level of consumption.

The rationale of Corollary 1 is consistent with many practices. Game levels are often designed such that it is more difficult for a high-level player to gain one more level than for a low-level player to do the same (thus, high-level players must worker harder to stay ahead of the game). In the Body-for-LIFE Challenge, performance is defined such that it is affected more by contestants' efforts than by how athletic they are at the beginning. The ways to achieve a high marginal rate of substitution of consumption for skill are limited only by imagination.

However, there is a caveat. No conclusion should be drawn about the limit case in which every consumer has the same skill level. For one reason, our equilibrium analysis and results on contest designs are based on differentiated
skills; they are no longer valid when consumers have the same skill level. 12 More important, our model is based on the premise that skill plays an indispensable role in determining a consumer's performance. In the limit case, the role of skill is absent, and other factors, such as noise in performance, may become important, implying that we may need a different model for these cases.

## Segmentation of the Population

Real-world contests often segment participants into smaller groups. For example, the National Basketball Association holds regional conferences, and some Mobile Millionaire operators classify players into different levels according to experience and prior performance. Whether these segmentation strategies generate more profit for marketers is a question that we explore in this subsection.

We consider the follow way of segmentation: dividing consumers into groups according to their skill factors, which we call "vertical segmentation." ${ }^{13}$ A vertical segmentation strategy causes the skill distribution for each segment to be different from that of the whole population. Consequently, two forces are at play. First, the size of each segment is smaller, which may negatively affect the marketer's profits $\left(\mathrm{P}_{6}\right)$. Second, the skill distribution in each segment may be less dispersive than the original skill distribution, which may positively affect the marketer's profits $\left(\mathrm{P}_{7}\right)$. The overall effect of a vertical segmentation depends on the exact skill distribution and how it is segmented. The following example shows that vertical segmentation can raise the marketer's overall expected profit:

Example 5 (vertical segmentation): Consider a 20-person consumer contest with the same skill distribution as in Example 2. According to Table 2, the marketer will pick an optimal prize sum of 1.53 and receive an expected profit of 1.17 . Consider that the firm keeps the same prize sum but runs two contests instead: one for those whose skill levels are within $[1,2]$ and the other for those whose skill levels are within $[9,10]$. Let x be the prize sum allocated to the first contest and $1.53-\mathrm{x}$ be the prize sum allocated to the second one. For example, if there are 17 consumers in the former contest and 3 in the latter, the expected revenues from two contests are $1.943 x$ and $1.949(1.53-x)$, respectively; thus the total profit is $1.812-.006 \mathrm{x}$, and the firm is better off with the vertical segmentation, no matter how it allocates its total prize sum between these two contests.

We summarize this discussion about vertical and horizontal segmentation in the following corollary:

Corollary 2 (vertical segmentation): The marketer can gain higher profit through vertical segmentation.

[^8]
## Managerial Implications

Although there is extensive literature on various types of contests, such as sales contests, R\&D tournaments, and sports contests, a popular type of contest-the consumer contest-remains underresearched. Consumer contests differ prominently from other contests in that the designers of consumer contests value contestants' consumption (how many minutes they spend on a mobile game) rather than their performance (how high their scores are). Consumer contests distinguish consumers with different skill levels; a higher-skilled consumer can achieve a better performance than a lower-skilled one even if both have the same consumption level. Therefore, consumer contests differ from other types of prize promotions, including games of chance, sweepstakes, and lotterylike contests, in which skill plays a minimum or no role.

This article offers several managerial implications regarding the design of consumer contests. A key decision facing a consumer contest designer is how to divide the prize sum. Should the prize sum be divided into multiple prizes, or should it be offered in a lump sum to the top performer (i.e., a winner-take-all design)? We find that this decision is closely related to the skill distribution among consumers. Specifically, if the skill distribution possesses the IHR property, the designer should choose a winner-takeall structure. A distribution has the IHR property if its density function is single peaked and declines no faster than the exponential rate. Intuitively, the IHR property implies that the higher a consumer's skill level, the more likely he or she will face opponents of similar skills than opponents of higher skills. Because facing opponents of similar skills encourages competition (and facing opponents who are too good discourages competition), the IHR property suggests that high-skilled consumers compete (and thus consume) more aggressively than low-skilled ones. Therefore, from the designer's point of view, high-skilled consumers are a more profitable segment. This segment is best motivated by the top prize; therefore, the designer should allocate the entire prize budget to the top prize.

The size of the contest is another factor to consider when choosing a prize structure. When the contest size is large, low-skilled contestants' chances of winning and their contribution to the designer's total profits are too low to be consequential. Therefore, only the high end of the skill distribution matters to the prize structure decision. Thus, in large contests, as long as the high end of the skill distribution has the IHR property, the designer should choose a winner-take-all design.

An increase in contest size can also lead to an increase in the designer's profit. Increasing the number of contestants has two opposite effects. On the one hand, it broadens a consumer contest's base (in terms of the number of consumers), and on the other hand, it reduces the promotional effect on each individual consumer (because each consumer now has a smaller chance of winning). If the skill distribution has the IHR property, the overall effect is positive. Intuitively, this is because in large contests, prizes are more likely to go to high-skilled consumers, and with the IHR property, high-skilled consumers generate higher revenue per unit prizes.

A novel finding of this research is that the effectiveness of a consumer contest (in terms of induced consumption) is closely linked to the dispersiveness of the skill distribution. Dispersiveness is a measure of inequality of skills among consumers. A less dispersive skill distribution means that a consumer is more likely to face competition from similarly skilled consumers than from higher-skilled consumers; thus, the consumer will compete more aggressively and consume more. As a result, less dispersive skill distribution leads to higher consumption across all skill levels.

The beneficial effect of having a less dispersive skill distribution has several practical implications for contest design. The contest designer can achieve a less dispersive distribution using two general approaches. The first approach involves selecting, grouping, and segmenting consumers. For example, the designer may benefit from segmenting contestants according to their skill levels so that each segment has a less dispersive skill distribution (as in Example 5). This seems to be consistent with what we find in the Mobile Millionaire example, in which providers that segment players into gold, silver, and bronze groups according to their demonstrated skills do better than those that do not. The dispersiveness result also implies that the contest designer may benefit from excluding low-skilled contestants and/or high-skilled contestants in some cases.

A second approach to achieve a less dispersive skill distribution involves adopting a performance evaluation scheme that handicaps high-skilled contestants (so that the competitive pressure across high- and low-skilled contestants is intensified). For example, by evaluating high-skilled contestants against a higher benchmark (as in the Body-forLIFE Challenge example), the designer can reduce the advantage of high-skilled contestants.

## Limitations and Further Research

A consumer contest is only one of the many sales promotion techniques. This study answers the question of how to best design the consumer contest, provided that a marketer already decides to adopt a consumer contest. Nevertheless, in practice, a marketer may face the question whether to adopt a consumer contest or to use alternative sales promotion techniques, such as quantity discounts. A consumer contest and other promotion techniques may not be mutually exclusive. This is because consumer contests taps into consumers' skill factors, whereas a majority of other promotions work on consumers' deal proneness. If consumer contests are used along with other promotions, the issue of how to coordinate among them becomes important. These issues remain the subject for further research.

This research focuses on consumer contests with a single goal: to maximize aggregate consumption or to maximize participation. We show that participation maximization often requires different prize structures than consumption maximization; specifically, the former often requires the prize sum to be divided into many small prizes, whereas the latter requires a winner-take-all structure. In practice, contest designers may pursue and balance among multiple goals at the same time. This raises the question of how to design a consumer contest when the designers have mixed
goals. For example, would it be better to offer a few large prizes and many small "participation" prize when the designer values both participation and aggregate consumption? Examining such issues may help explain the large variety of consumer contest designs observed in practice.

Another venue for further research is to test our theoretical predictions regarding contest designs in laboratory experimental settings. Experimental settings provide researchers excellent control of contest designs in terms of prize structure, performance function, and skill distribution. Such control is rarely available in field settings. Prior experimental research on contest designs has examined contests with identical skill levels (e.g., Orrison, Schotter, and Weigelt 2004) but not contests with different skill levels.

## Appendix

## Order Statistics Used in the Article

Equation 6 comes directly from the definition of $F_{n-j: n-1}(\mu)$ and the notion that the probability of $\mu$ being $l$ th largest among $\mathrm{n}-1$ samples is

$$
\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1} .
$$

For Equation 7, note that

$$
\begin{aligned}
& \int_{0}^{\mu} j\binom{n-1}{n-j-1} F(\sigma)^{n-j-1}[1-\mathrm{F}(\sigma)]^{j-1} \mathrm{f}(\sigma) \mathrm{d} \sigma \\
= & \int_{0}^{\mathrm{F}(\mu)} \mathrm{j}\binom{\mathrm{n}-1}{\mathrm{n}-\mathrm{j}-1} \mathrm{~s}^{\mathrm{n}-\mathrm{j}-1}(1-\mathrm{s})^{\mathrm{j}-1} \mathrm{ds} \\
= & \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1}
\end{aligned}
$$

(by repeated integration by parts).
We also define $\mathrm{F}_{\mathrm{n}: \mathrm{n}-1}(\mu)=0, \mathrm{~F}_{\mathrm{n}-\mathrm{n}: \mathrm{n}-1}(\mu)=1, \mathrm{f}_{\mathrm{n}: \mathrm{n}-1}(\mu)=$ 0 , and $\mathrm{f}_{\mathrm{n}-\mathrm{n}: \mathrm{n}-1}(\mu)=0$.

## Proof of $\mathbf{P}_{\mathbf{1}}$

Step 1. Let $\mathrm{x}(\mu)$ denote the equilibrium performance, and assume that $x(\mu)$ is strictly increasing (we confirm this in Step 2). Let $U(\mu, x)$ denote the expected utility when a consumer's skill is $\mu$ and the consumer chooses performance $x$. When every other consumer plays according to $x(\mu)$, the consumer's probability of winning at least jth place is $F_{n-j: n-1}\left(x^{-1}(x)\right)$, where $x^{-1}(\cdot)$ is the inverse of $x(\cdot)$ :

$$
U(\mu, x)=\sum_{j=1}^{n} \delta_{j} \frac{1}{j}-F_{n-j: n-1}\left(x^{-1}(x)\right)-(p-\theta) \frac{x}{\mu} .
$$

For $x(\mu)$ to be an optimal strategy, it must be that

$$
\begin{gather*}
\frac{\partial U(\mu, x)}{\partial x}=\sum_{j=1}^{n} \delta_{j}{ }_{j}^{1} f_{\mathrm{f}-\mathrm{j}: \mathrm{n}-1}\left(\mathrm{x}^{-1}(\mathrm{x})\right)  \tag{A1}\\
\frac{d x^{-1}(\mathrm{x})}{\mathrm{dx}}-(\mathrm{p}-\theta) \frac{1}{\mu}=0
\end{gather*}
$$

In a symmetric-strategy equilibrium, $x^{-1}(x)=\mu$. Substituting $x^{-1}(x)=\mu$ into Equation A1 and rearranging terms yields

$$
\begin{equation*}
\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}} \frac{1}{\mathrm{j}} \mathrm{f}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) \mu \mathrm{d} \mu=(\mathrm{p}-\theta) \mathrm{dx} \tag{A2}
\end{equation*}
$$

because the lowest-skilled consumer always chooses zero performance in equilibrium, $x(\mu)=0$. Applying this boundary condition, we can obtain the solution for Equation A2 as

$$
x(\mu)=\frac{1}{p-\theta} \sum_{j=1}^{n}\left[\delta_{j} \int_{\underline{\mu}}^{\mu}{ }_{j} \underline{j}_{n-j: n-1}(\sigma) \sigma d \sigma\right]
$$

It follows that

$$
t(\mu)=\frac{1}{p-\theta} \sum_{j=1}^{n}\left[\delta_{j} \frac{1}{\mu} \int_{\underline{\mu}}^{\mu}-\frac{f_{j}}{f_{n-j: n-1}}(\sigma) \sigma d \sigma\right] .
$$

Step 2 . We now verify that $\mathrm{x}(\mu)$ is strictly increasing. This is evident because $\delta_{\mathrm{j}} \geq 0$ (inequality holds for at least one $j$ ) and $\int_{\mu}^{\mu} f_{n-j: n-1}(\sigma) \sigma d \sigma / j \geq 0$ for any $\mu$ and $j$.

Step 3. We now show that $\mathrm{x}(\mu)$ is also sufficient to maximize $U(\mu, x)$. We assume that a consumer with skill $\mu$ chooses an arbitrary performance, denoted as $\mathrm{x}\left(\mu^{\prime}\right)$. Note that

$$
\frac{\partial \mathrm{U}\left(\mu, \mathrm{x}\left(\mu^{\prime}\right)\right)}{\partial \mu^{\prime}}=\mathrm{U}_{\mathrm{x}}\left(\mu, \mathrm{x}\left(\mu^{\prime}\right)\right) \mathrm{x}^{\prime}\left(\mu^{\prime}\right)
$$

Because $\mathrm{x}^{\prime}(\cdot)>0$, the sign of the preceding equation is determined by $U_{x}\left(\mu, x\left(\mu^{\prime}\right)\right)$. In line with the first-order condition (Equation $A 1$ ), $\mathrm{U}_{\mathrm{x}}\left(\mu, \mathrm{x}\left(\mu^{\prime}\right)\right)=0$ at $\mu^{\prime}=\mu$. Furthermore,

$$
\mathrm{U}_{\mathrm{x}}\left(\mu, \mathrm{x}\left(\mu^{\prime}\right)\right)=\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}}^{\mathrm{j}} \mathrm{j}_{\mathrm{n}}^{1} \mathrm{f}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}\left(\mu^{\prime}\right) \frac{\mathrm{dx}-1\left(\mathrm{x}\left(\mu^{\prime}\right)\right)}{\mathrm{dx}}-(\mathrm{p}-\mathrm{b}) \frac{1}{\mu}
$$

is positive for any $\mu>\mu^{\prime}$ and negative for any $\mu<\mu^{\prime}$. Therefore, $\mu^{\prime}=\mu$ is a global maximum for $U\left(\mu, x\left(\mu^{\prime}\right)\right)$. Because this holds for every $\mu$, we conclude that $x(\mu)$ is optimal.

## Proof of $P_{2}$

For $\mathrm{P}_{2 \mathrm{a}}$, we pick $\mathrm{j}^{*}$ such that $\varepsilon_{j^{*}}=\max \left\{\varepsilon_{1}, \varepsilon_{2}, \ldots, \varepsilon_{\mathrm{n}-1}\right\}$. From Equation 11, it is straightforward that given any prize $\operatorname{sum} \mathrm{V}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{j}}$, the optimal strategy for the marketer is to set $\delta_{i}=0$ for all $\mathrm{i} \neq \mathrm{j}^{*}$. This is equivalent to setting $\mathrm{v}_{1}=$ $\mathrm{v}_{2}=\ldots=\mathrm{v}_{\mathrm{j}^{*}}=\mathrm{V} / \mathrm{j}^{*}$, and $\mathrm{v}_{\mathrm{j}^{*}+1}=\mathrm{v}_{\mathrm{j}^{*}+2}=\ldots=\mathrm{v}_{\mathrm{n}-1}=0$.

For $\mathrm{P}_{2 \mathrm{~b}}$, the marketer's optimization problem is simplified to $\max _{\mathrm{V}}\left\{\mathrm{V} \varepsilon_{\mathrm{j}^{*}}-\mathrm{c}(\mathrm{V})\right\}$. Thus, $\mathrm{V}^{*}$ is the solution to $\varepsilon_{\mathrm{j}^{*}}=$ $c^{\prime}\left(V^{*}\right)$, if $c^{\prime}(0)<\varepsilon_{j^{*}}$, and zero if otherwise.

## Proof of $P_{3}$ and $P_{\mathbf{4}}$

Lemma A1: $\mathrm{f}(\mathrm{x}), \mathrm{g}(\mathrm{x})$ are continuous functions defined on $[a, b]$. Furthermore, $\int_{a}^{b} f(x) d x=0$, and $f(x)$ single-crosses zero (first negative and then positive). Let the crossing point be $\xi$. We have $\int_{a}^{b} f(x) g(x) d x>0$ if either (a) or (b) is satisfied:
(a) $\mathrm{g}^{\prime}(\mathrm{x})>0$, on $[\mathrm{a}, \mathrm{b}]$, and
(b) $\mathrm{g}(\mathrm{x}) \leq \mathrm{g}(\xi)$ for $\mathrm{x} \leq \xi$, and $\mathrm{g}^{\prime}(\mathrm{x})>0$ for $\mathrm{x}>\xi$.

Proof. Because Condition a implies Condition b, we show only Condition b:

$$
\begin{aligned}
\int_{a}^{b} f(x) g(x) d x & =\int_{a}^{\xi} f(x) g(x) d x+\int_{\xi}^{b} f(x) g(x) d x \\
& >g(\xi) \int_{a}^{\xi} f(x) d x+g(\xi) \int_{\xi}^{b} f(x) d x \\
& =g(\xi) \int_{a}^{b} f(x) d x=0 .
\end{aligned}
$$

Lemma A2:
(a) $\int_{\mu}^{\bar{\mu}} n F_{n-j: n-1}(\mu) f(\mu) d \mu / j=1$, for $j=1,2, \ldots, n$.
(b) $\overline{\mathrm{n}} \mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) / \mathrm{j}-\mathrm{n} \mathrm{F}_{\mathrm{n}-\mathrm{k}: \mathrm{n}-1}(\mu) / \mathrm{k}$ single-crosses zero, for $1 \leq \mathrm{j}<\mathrm{k}<\mathrm{n}$.
(c) $n \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) / \mathrm{j}-(\mathrm{n}+1) \mathrm{F}_{\mathrm{n}+1-\mathrm{j}: \mathrm{n}}(\mu) / \mathrm{j}$ single-crosses zero, for $\mathrm{j}=1,2, \ldots, \mathrm{n}-1$.

## Proof.

(a) $\int_{\underline{\mu}}^{\bar{\mu}} \frac{\mathrm{n}}{\mathrm{j}} \mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) \mathrm{f}(\mu) \mathrm{d} \mu$

$$
\begin{aligned}
& =\frac{\mathrm{n}}{\mathrm{j}} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1} \int_{\underline{\mu}}^{\bar{\mu}}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1} \mathrm{f}(\mu) \mathrm{d} \mu \\
& =\frac{\mathrm{n}}{\mathrm{j}} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1} \int_{0}^{1}\binom{\mathrm{n}-1}{l} \mathrm{x}^{l}(1-\mathrm{x})^{\mathrm{n}-l-1} \mathrm{dx} \\
& =\frac{\mathrm{n}}{\mathrm{j}} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1} \frac{1}{\mathrm{n}} \text { (due to repeated integration by parts) } \\
& =1 .
\end{aligned}
$$

(b) $\underset{j}{\frac{1}{j}} \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu)-\frac{1}{\mathrm{j}+1} \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu)$

$$
\begin{aligned}
= & \frac{1}{\mathrm{j}} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1} \\
& -\frac{1}{\mathrm{j}+1} \sum_{l=\mathrm{n}-\mathrm{j}-1}^{\mathrm{n}-1}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1} \\
= & \frac{1}{\mathrm{j}(\mathrm{j}+1)} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1}\binom{\mathrm{n}-1}{l} \mathrm{~F}(\mu)^{l}[1-\mathrm{F}(\mu)]^{\mathrm{n}-l-1} \\
& -\frac{1}{\mathrm{j}+1}\binom{\mathrm{n}-1}{\mathrm{n}-\mathrm{j}-1} \mathrm{~F}(\mu)^{\mathrm{n}-\mathrm{j}-1}[1-\mathrm{F}(\mu)]^{\mathrm{j}} \\
= & \frac{1}{\mathrm{j}+1}\binom{\mathrm{n}-1}{\mathrm{n}-\mathrm{j}-1} \mathrm{~F}(\mu)^{\mathrm{n}-\mathrm{j}-1}[1-\mathrm{F}(\mu)]^{\mathrm{j}} \\
& \left\{\begin{array}{c}
\mathrm{l} \\
\mathrm{j}\binom{\mathrm{n}-1}{\mathrm{n}-\mathrm{j}-1}^{-1} \sum_{l=\mathrm{n}-\mathrm{j}}^{\mathrm{n}-1}\binom{\mathrm{n}-1}{l} \\
\\
\\
{\left[\frac{\mathrm{~F}(\mu)}{1-\mathrm{F}(\mu)}\right]^{l+\mathrm{j}+1-\mathrm{n}}-1}
\end{array}\right.
\end{aligned}
$$

The sign in Proof $b$ is determined by the sign of the term in braces, which goes from negative to positive. Thus, $\mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) / \mathrm{j}-\mathrm{F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\mu) /(\mathrm{j}+1)$ single-crosses zero. Using Proof a, we can further conclude that $n F_{n-j: n-1}(\sigma) f(\sigma) / j$ first-order stochastically dominates $n F_{n-j-1: n-1}(\sigma) f(\sigma) /(j+1)$. Because of the transitive property of first-order stochastic dominance, $\mathrm{nF}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma) \mathrm{f}(\sigma) / \mathrm{j}$ first-order stochastically dominates $\mathrm{nF}_{\mathrm{n}-\mathrm{k}: \mathrm{n}-1}(\sigma) \mathrm{f}(\sigma) / \mathrm{k}$ for any $\mathrm{k}>\mathrm{j}$. Thus, we have Lemma 2(b).

Finally,
(c) $\frac{d}{d \mu}\left[\frac{n+1}{j} F_{n+1-j: n}(\mu)-\frac{1}{j} F_{n-j: n-1}(\mu)\right]$

$$
=j\binom{n}{n-j} F(\mu)^{n-j}[1-F(\mu)]^{j-1} f(\mu)
$$

$$
-j\binom{n-1}{n-j-1} F(\mu)^{n-j-1}[1-F(\mu)]^{j-1} f(\mu)
$$

$$
=j\binom{n-1}{n-j-1} F(\mu)^{n-j-1}[1-F(\mu)]^{j-1}
$$

$$
\mathrm{f}(\mu)\left[\frac{\mathrm{n}}{\mathrm{n}-\mathrm{j}} \mathrm{~F}(\mu)-1\right]
$$

In the last line, the sign of the final term in parentheses (i.e., $[\mathrm{n} /(\mathrm{n}-\mathrm{j})] \mathrm{F}(\mu)-1$ ) goes from negative to positive, and so does that of the entire term. Because $(n+1) F_{n+1-j: n}(\mu) /$ $j-F_{n-j: n-1}(\mu) / j=0$, we know that $(n+1) F_{n+1-j: n}(\mu) / j-$ $F_{n-j: n-1}(\mu) / j$ is first negative and then positive; note that $(n+1) F_{n-j+1: n}(\bar{\mu}) / j-F_{n-j: n-1}(\bar{\mu}) / j=n / j>0$.

Proof of $P_{3}$. We prove $P_{3}$ by showing that $\varepsilon_{1}>\varepsilon_{2}>\ldots>$ $\varepsilon_{\mathrm{n}}$. Because in equilibrium, $\mathrm{v}_{\mathrm{n}}=0$, if $\mathrm{n}=2$, we always have a winner-take-all structure. Hereinafter, we consider only the case in which $\mathrm{n} \geq 3$.
(A3) $\varepsilon_{j}=\frac{n p}{p-\theta} \int_{\underline{\mu}}^{\bar{\mu}} \frac{1}{\mu} \int_{\underline{\mu}}^{\mu} \sigma \underset{{ }_{j}}{1} f_{n-j: n-1}(\sigma) \mathrm{d} \sigma f(\mu) \mathrm{d} \mu$

Thus, for $\mathrm{j}=1,2, \ldots, \mathrm{n}-1$,

$$
\begin{gathered}
\varepsilon_{\mathrm{j}}-\varepsilon_{\mathrm{j}+1}=\frac{\mathrm{p}}{\mathrm{p}-\theta} \int_{\underline{\mu}}^{\bar{\mu}}\left[\frac{n}{\mathrm{n}} \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma)-\frac{\mathrm{n}}{\mathrm{j}+1} \mathrm{~F}_{\mathrm{n}+1-\mathrm{j}: \mathrm{n}-1}(\sigma)\right] \\
\mathrm{f}(\sigma)\left[-\frac{1}{\mathrm{f}(\sigma)} \int_{\mu}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu\right] \mathrm{d} \sigma .
\end{gathered}
$$

$$
\begin{aligned}
& =\frac{n p}{p-\theta} \int_{\underline{\mu}}^{\bar{\mu}} \sigma \underset{j}{1} f_{n-j: n-1}(\sigma) \int_{\sigma}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu \mathrm{~d} \sigma \\
& =\frac{n p}{p-\theta}\left\{\left.\frac{1}{j} \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma) \sigma \int_{\sigma}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu\right|_{\underline{\mu}} ^{\bar{\mu}}\right. \\
& \left.-\int_{\underline{\mu}}^{\bar{\mu}}{ }_{\underline{1}}{ }_{\mathrm{j}}^{\mathrm{F}} \mathrm{~F}_{\mathrm{n}-\mathrm{j}: \mathrm{n}-1}(\sigma)\left[\int_{\sigma}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu-\mathrm{f}(\sigma)\right] \mathrm{d} \sigma\right\} \\
& =\frac{p}{p-\theta} \int_{\underline{\mu}}^{\bar{\mu}} \frac{n}{j} F_{n-j: n-1}(\sigma) f(\sigma) \\
& {\left[1-\frac{1}{\mathrm{f}(\sigma)} \int_{\sigma}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu\right] \mathrm{d} \sigma .}
\end{aligned}
$$

In line with Lemmas A2a and A2b and Lemma A1a, we know that a sufficient condition for $\varepsilon_{j}>\varepsilon_{j+1}$ is that $\mathrm{H}(\sigma) \triangleq-\int_{\sigma}^{\bar{\mu}}[\mathrm{f}(\mu) / \mu] \mathrm{d} \mu / \mathrm{f}(\sigma)$ is monotonic. Next, we show that $\mathrm{H}^{\prime}(\sigma)>0$ holds under the IHR condition:

$$
\mathrm{H}^{\prime}(\sigma)=\frac{1}{\mathrm{f}(\sigma)^{2} \sigma}\left[\mathrm{f}^{\prime}(\sigma) \sigma \int_{\sigma}^{\bar{\mu}} \frac{1}{\mu} \mathrm{f}(\mu) \mathrm{d} \mu+\mathrm{f}(\sigma)^{2}\right] .
$$

When $\mathrm{f}^{\prime}(\sigma) \geq 0, \mathrm{H}^{\prime}(\sigma)>0$ holds naturally. When $\mathrm{f}^{\prime}(\sigma)<0$,

$$
\begin{aligned}
\mathrm{H}^{\prime}(\sigma) & >\frac{1}{\mathrm{f}(\sigma)^{2} \sigma}\left\{\mathrm{f}^{\prime}(\sigma)[1-\mathrm{F}(\sigma)]+\mathrm{f}(\sigma)^{2}\right\} \\
& =\frac{[1-\mathrm{F}(\sigma)]^{2}}{\mathrm{f}(\sigma)^{2} \sigma} \frac{\mathrm{~d}}{\mathrm{~d} \sigma}\left[\frac{\mathrm{f}(\sigma)}{1-\mathrm{F}(\sigma)}\right]>0,
\end{aligned}
$$

where the last step is due to the IHR condition. Thus, we have $\varepsilon_{1}>\varepsilon_{2}>\ldots>\varepsilon_{\mathrm{n}}$.

Proof of $P_{4}$. We prove $P_{4}$ by showing that $\varepsilon_{1}>\varepsilon_{k}, k=2$, $3, \ldots, \mathrm{n}-1$. Given that the skill distribution has the pseudoIHR property with starting point $\chi$, in line with Lemma A 1 b , a sufficient condition for $\varepsilon_{1}>\varepsilon_{\mathrm{k}}, \mathrm{k}=2,3, \ldots, \mathrm{n}-1$ is

$$
\begin{align*}
\eta(\chi ; \mathrm{k}, \mathrm{n}) & \triangleq \mathrm{F}_{1: \mathrm{n}-1}(\chi)-\frac{1}{\mathrm{k}} \mathrm{~F}_{\mathrm{n}-\mathrm{k}: \mathrm{n}-1}(\chi) \leq 0,  \tag{A4}\\
& \text { for all } \mathrm{k}=2,3, \ldots, \mathrm{n}-1 .
\end{align*}
$$

When $\chi=\underline{\mu}$, Equation A4 always holds. When $\chi>\underline{\mu}$,

$$
\eta(\chi ; \mathrm{k}, \mathrm{n})=\mathrm{F}(\chi)^{\mathrm{n}-1}\left\{1-\frac{1}{\mathrm{k}} \sum_{l=1}^{\mathrm{k}}\binom{\mathrm{n}-1}{l-1}\left[\frac{1-\mathrm{F}(\chi)}{\mathrm{F}(\chi)}\right]^{l-1}\right\} .
$$

Let

$$
\beta_{l} \triangleq\binom{\mathrm{n}-1}{l-1}\left[\frac{1-\mathrm{F}(\chi)}{\mathrm{F}(\chi)}\right]^{l-1}
$$

A sufficient condition for Equation A4 is

$$
\begin{equation*}
\frac{1}{\mathrm{k}} \sum_{l=1}^{\mathrm{k}} \beta_{l}>1, \text { for all } \mathrm{k}=2,3, \ldots, \mathrm{n}-1 . \tag{A5}
\end{equation*}
$$

Note that the sequence $\beta_{1}, \beta_{2}, \ldots, \beta_{\mathrm{n}-1}$ is single peaked, which implies that $\sum_{l=1}^{\mathrm{k}} \beta_{l} / \mathrm{k}$ reaches its minimum either at $\mathrm{k}=1$ or at $\mathrm{k}=\mathrm{n}-1$. Because $\beta_{l}=1$, a necessary and sufficient condition for Equation A5 is
(A6) $\frac{1}{\mathrm{n}-1} \sum_{l=1}^{\mathrm{n}-1} \beta_{l}=\frac{1}{\mathrm{n}-1}\left\{\left[\frac{1}{\mathrm{~F}(\chi)}\right]^{\mathrm{n}-1}-\left[\frac{1-\mathrm{F}(\chi)}{\mathrm{F}(\chi)}\right]^{\mathrm{n}-1}\right\}>1$.
$\mathrm{P}_{4}$ follows because $\sum_{l=1}^{\mathrm{n}-1} \beta_{l} /(\mathrm{n}-1)$ increases in n .

## Proof of $\mathrm{P}_{5}$

$$
\begin{aligned}
P_{j}(u) & =\frac{1}{j} F_{n-j: n-1}\left(F^{-1}(u)\right)=\frac{1}{j} \int_{\underline{\mu}}^{F^{-1}(u)} f_{n-j: n-1}(\mu) d \mu \\
& =\frac{1}{j} \int_{\underline{\mu}}^{F-1(u)} j\binom{n-1}{n-j-1} F(\mu)^{n-j-1[1-F(\mu)] j-1 f(\mu) d \mu}
\end{aligned}
$$

$$
=\int_{0}^{\mathrm{u}}\binom{\mathrm{n}-1}{\mathrm{n}-\mathrm{j}-1} \sigma^{\mathrm{n}-\mathrm{j}-1}(1-\sigma)^{\mathrm{j}-1} \mathrm{~d} \sigma .
$$

The cutoff skill quantile is given by

$$
\begin{equation*}
\sum_{j=1}^{n-1} \delta_{j} P_{j}(u)=e \tag{A7}
\end{equation*}
$$

Suppose that $u^{*}$ exists (we verify this subsequently), and let $\mathrm{j}_{\mathrm{u}^{*}}=\arg \max _{\mathrm{j}}\left\{\mathrm{P}_{\mathrm{j}}\left(\mathrm{u}^{*}\right)\right\}$. Thus, for optimality, it must be that $\delta_{\mathrm{j}}=\mathrm{V}$ for $\mathrm{j}=\mathrm{j}_{\mathrm{u}^{*}}$ and $\delta_{\mathrm{j}}=0$ for $\mathrm{j} \neq \mathrm{j}_{\mathrm{u}^{*}}$; otherwise, the marketer could allocate a greater amount to the $\mathrm{j}_{\mathrm{u}}$ th prize and thus find a lower $\mathrm{u}^{*}$; this would be a contradiction. Therefore, $\overline{\mathrm{P}}\left(\mathrm{u}^{*}\right)=\mathrm{e} / \mathrm{V}$, and the prize sum should be split evenly among the first $\mathrm{j}_{\mathrm{u}^{*}}$ prizes. It always has a solution because $0 \leq \mathrm{e} / \mathrm{V}<1$, and $\overline{\mathrm{P}}(\mathrm{u})$ is a continuous function with $\overline{\mathrm{P}}(0)=0$ and $\overline{\mathrm{P}}(1)=1$.

Because condition $\overline{\mathrm{P}}\left(\mathrm{u}^{*}\right)=\mathrm{e} / \mathrm{V}$ is independent of $\mathrm{F}(\mu)$, $\mathrm{j}_{\mathrm{u}^{*}}$ is independent of $\mathrm{F}(\mu)$. Because $\overline{\mathrm{P}}(\mathrm{u})$ is a weakly increasing function, $\overline{\mathrm{P}}\left(\mathrm{u}^{*}\right)=\mathrm{e} / \mathrm{V}$ implies that $\mathrm{u}^{*}$ (weakly) increases in e/V. To show that $\mathrm{j}_{\mathrm{u}^{*}}$ (weakly) decreases in e/V, it suffices to show that $\mathrm{j}_{\mathrm{u}^{*}}$ (weakly) decreases in $\mathrm{u}^{*}$ for any $u^{*}$.

In line with Lemma $A 2 b$ and the notion that $P_{j}(u)=$ $F_{n-j: n-1}\left(F^{-1}(\mu)\right) / j, P_{j}(u)-P_{k}(u)(k>j)$ single-crosses zero from negative to positive. Thus, when $u^{*}$ increases, it is impossible for $\mathrm{j}_{\mathrm{u}^{*}}=\arg \max _{\mathrm{j}}\left\{\mathrm{P}_{\mathrm{j}}\left(\mathrm{u}^{*}\right)\right\}$ to increase.

## Proof of $\boldsymbol{P}_{\mathbf{6}}$

We hold constant the prize structure $\left(\delta_{1}, \delta_{2}, \ldots, \delta_{n-1}, 0\right)$ and consider the marketer's expected profit from an nconsumer contest, $\pi_{n}$, and from an ( $n+1$ )-consumer contest, $\pi_{\mathrm{n}+1}$ :

$$
\begin{gathered}
\pi_{n+1}-\pi_{n}=\frac{p}{p-\theta} \sum_{j=1}^{n-1} \delta_{j} \\
\int_{\underline{\mu}}^{\bar{\mu}}\left[\frac{n+1}{j} F_{n-j+1: n}(\sigma)-\frac{n}{j} F_{n-j: n-1}(\sigma)\right] f(\sigma) H(\sigma) d \sigma,
\end{gathered}
$$

where $\mathrm{H}(\sigma)$ is defined as in the proof of $\mathrm{P}_{3}$ and $\mathrm{P}_{4}$. In line with Lemmas A2a and A2c, we know that $\int_{\underline{\mu}}^{\bar{\mu}}$ $\left[(n+1) F_{n-j+1: n}(\sigma) / j-n F_{n-j: n-1}(\sigma) / j\right] f(\sigma) d \sigma=0$ and that $(n+1) F_{n-j+1: n}(\sigma) / j-n F_{n-j: n-1}(\sigma) / j$ single-crosses zero. If we apply Lemma A1a, a strictly increasing $H(\sigma)$ is sufficient for $\pi_{n+1}-\pi_{n}>0$. As in the proof of $\mathrm{P}_{4}$, we can show that $\pi_{\mathrm{n}+1}-\pi_{\mathrm{n}}>0$ also holds when F has the pseudo-IHR property and n is sufficiently large.

## Proof of $P_{7}$

Consider two consumer contests that are the same, except one has skill distribution F and the other has skill distribution G. Let $u$ denote a quantile; then, $\mathrm{F}^{-1}(\mathrm{u})$ and $\mathrm{G}^{-1}(\mathrm{u})$ are corresponding skill levels in distribution F and G . Let $\mathrm{t}_{\mathrm{F}}^{\mathrm{q}}(\mathrm{u})=\mathrm{t}\left(\mathrm{F}^{-1}(\mathrm{u})\right)$ denote the equilibrium consumption at quantile $u$ in a consumer contest with skill distribution $F$. As such, $\mathrm{t}_{\mathrm{G}}^{\mathrm{q}}(\mathrm{u})$ is similarly defined for the consumer contest
with skill distribution G. $\mathrm{P}_{7}$ can be formally presented as follows:

F is less dispersive than $\mathrm{G} \Rightarrow \mathrm{t}_{\mathrm{F}}^{\mathrm{q}}(\mathrm{u}) \geq \mathrm{t}_{\mathrm{G}}^{\mathrm{q}}(\mathrm{u})$, for every $0 \leq \mathrm{u} \leq 1$.
We can rewrite the equilibrium consumption (Equation 8) as follows:

$$
\begin{aligned}
\mathrm{t}_{\mathrm{F}}^{\mathrm{q}}(\mathrm{u})= & \frac{1}{\mathrm{p}-\theta} \sum_{\mathrm{j}=1}^{\mathrm{n}-1} \delta_{j} \int_{0}^{\mathrm{u}} \frac{\mathrm{~F}^{-1}(\mathrm{~s})}{\mathrm{F}^{-1}(\mathrm{u})}\binom{\mathrm{n}-1}{j} \\
& \mathrm{~s}^{\mathrm{n}-\mathrm{j}-1}(1-\mathrm{s}) \mathrm{j}-1 \mathrm{ds}, \text { and } \\
\mathrm{t}_{\mathrm{F}}^{\mathrm{q}}(\mathrm{u})-\mathrm{t}_{\mathrm{G}}^{\mathrm{q}}(\mathrm{u})= & \frac{1}{\mathrm{p}-\theta} \sum_{\mathrm{j}=1}^{\mathrm{n}-1} \delta_{j} \int_{0}^{\mathrm{u}}\left[\frac{\mathrm{~F}^{-1}(\mathrm{~s})}{\mathrm{F}^{-1}(\mathrm{u})}-\frac{\mathrm{G}^{-1}(\mathrm{~s})}{\mathrm{G}^{-1}(\mathrm{u})}\right]\binom{\mathrm{n}-1}{j} \\
& \mathrm{~s}^{\mathrm{n}-\mathrm{j}-1}(1-\mathrm{s}) \mathrm{j}^{\mathrm{j}-1} d \mathrm{ds} .
\end{aligned}
$$

A sufficient condition for $t_{F}^{q}(u)-t_{G}^{q}(u) \geq 0$ is

$$
\frac{\mathrm{F}^{-1}(\mathrm{u})}{\mathrm{F}^{-1}(\mathrm{~s})} \leq \frac{\mathrm{G}^{-1}(\mathrm{u})}{\mathrm{G}^{-1}(\mathrm{~s})}, \forall 0 \leq \mathrm{s}<\mathrm{u} \leq 1
$$

## Proof of Corollary 1

We consider two consumer contests, A and B , with performance functions $X^{A}(\mu, t)$ and $X^{B}(\mu, t)$, respectively. We
redefine $\phi(\mu)$ as consumers' skill factors in Contest B and let $G$ denote its distribution. The relationship between $G$ and $F$ is

$$
\begin{equation*}
\mathrm{G}^{-1}(\mathrm{u})=\phi\left(\mathrm{F}^{-1}(\mathrm{u})\right) \tag{A8}
\end{equation*}
$$

In line with $\mathrm{P}_{7}$, a sufficient condition for the Contest B to generate higher equilibrium consumption is

$$
\begin{equation*}
\frac{\mathrm{G}^{-1}(\mathrm{u})}{\mathrm{G}^{-1}(\mathrm{~s})} \leq \frac{\mathrm{F}^{-1}(\mathrm{u})}{\mathrm{F}^{-1}(\mathrm{~s})}, \forall 0 \leq \mathrm{s}<\mathrm{u} \leq 1 . \tag{A9}
\end{equation*}
$$

Let $\mu=\mathrm{F}^{-1}(\mathrm{u})$ and $\sigma=\mathrm{F}^{-1}(\mathrm{~s})$. In line with Equation A8, we can rewrite Equation A9 as
(A10)

$$
\frac{\mu}{\sigma} \geq \frac{\phi(\mu)}{\phi(\sigma)}, \underline{\mu} \leq \sigma<\mu \leq \bar{\mu} .
$$

An equivalent representation of Equation A 10 is (let $\sigma=$ $\mu+\Delta \mu$ and $\Delta \mu \rightarrow 0):$

$$
\phi^{\prime}(\mu) \mu-\phi(\mu) \leq 0 \text {, for all } \mu \in(\underline{\mu}, \bar{\mu}) .
$$

This condition holds if and only if

$$
\frac{\phi(\mu)}{t^{\prime}(\mu)}-\frac{\mu}{\mathrm{t}} \geq 0 \text {, for all } \mu \in(\underline{\mu}, \bar{\mu}),
$$

where $\mu / t$ is the marginal rate of substitution for Contest A and $\phi(\mu) /\left[t \phi^{\prime}(\mu)\right]$ for Contest B.

## REFERENCES

Basu, Amiya K., Rajiv Lal, V. Srinivasan, and Richard Staelin (1985), "Salesforce Compensation Plans: An Agency Theoretic Perspective," Marketing Science, 4 (4), 267-82.
Baye, Michael R. and Heidrun C. Hoppe (2003), "The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games," Games and Economic Behavior, 44 (2), 217-26.
-, Dan Kovenock, and Casper G. de Vries (1993), "Rigging the Lobbying Process: An Application of the All-Pay Auction," American Economic Review, 83 (1), 289-94.
Chandon, Pierre, Brian Wansink, and Gilles Laurent (2000), "A Benefit Congruency Framework of Sales Promotion Effectiveness," Journal of Marketing, 64 (October), 65-81.
Che, Yeon-Koo and Ian Gale (2003), "Optimal Design of Research Contests," American Economic Review, 93 (3), 646-71.
Clark, Derek J. and Christian Riis (1998), "Competition over More Than One Prize," American Economic Review, 88 (1), 276-89.
Dasgupta, P. and J. Stiglitz (1980), "Industrial-Structure and the Nature of Innovative Activity," Economic Journal, 90 (358), 266-93.
Feess, Eberhard and Gerd Muehlheusser (2003), "Transfer Fee Regulations in European Football," European Economic Review, 47 (4), 645-69.
Feinman, Jerry P., Robert D. Blashek, and Richard McCabe (1986), Sweepstakes, Prize Promotions, Games and Contests. Homewood, IL: Dow Jones/Richard D. Irwin.
Fullerton, Richard L. and R. Preston McAfee (1999), "Auctioning Entry into Tournaments," Journal of Political Economy, 107 (3), 573-605.

Gerstner, Eitan and James D. Hess (1995), "Pull Promotions and Channel Coordination," Marketing Science, 14 (1), 43-60.
Glazer, Amihai and Refael Hassin (1988), "Optimal Contests," Economic Inquiry, 16 (1), 133-43.

Hamman, Bob (2000), "Forget the Web Ads: Online Promos Capture Attention," Marketing News, 34 (23), 28-29.
Informa Telecoms and Media (2005), "Mobile Entertainment," Informa Telecoms and Media Report No: INFM1270224.
Iyer, Ganesh and Amit Pazgal (2003), "Internet Shopping Agents: Virtual Co-Location and Competition," Marketing Science, 22 (1), 85-106.

Kalra, Ajay and Mengze Shi (2001), "Designing Optimal Sales Contests: A Theoretical Perspective," Marketing Science, 20 (2), 170-93. _ and __ (2002), "Consumer Value-Maximizing Sweepstakes \& Contests: A Theoretical and Experimental Investigation," Review of Marketing Science Working Papers, 1 (3), 1-36.

Kopalle, Praveen K. and Scott A. Neslin (2003), "The Economic Viability of Frequency Reward Programs in a Strategic Competitive Environment," Review of Marketing Science, 1 (1), 1-39.

Kotler, Phillip (1999), Marketing Management: Analysis, Planning, Implementation, and Control. Upper Saddle River, NJ: Prentice Hall.
Krishna, Vijay and John Morgan (1998), "The Winner-Take-All Principle in Small Tournaments," in Advances in Applied Microeconomics, Vol. 7, Michael R. Baye, ed. Stamford, CT: JAI Press, 61-74.
Lazear, Edward P. and Sherwin Rosen (1981), "Rank-Order Tournaments as Optimum Labor Contracts," Journal of Political Economy, 89 (5), 841-64.
Liu, De, Xianjun Geng, and Andrew B. Whinston (2007), "Status Seeking and the Design of Online Entertainment Communities," in Managing in the Information Economy: Current Research Issues, Uday S. Karmarkar and Uday M. Apte, eds. New York: Kluwer Academic Publishers, 289-314.

Moldovanu, Benny and Aner Sela (2001), "The Optimal Allocation of Prizes in Contests," American Economic Review, 91 (3), 542-58.
Nalebuff, Barry J. and Joseph E. Stiglitz (1983), "Prizes and Incentives: Towards a General Theory of Compensation and Competition," Bell Journal of Economics, 14 (1), 21-43.
Neslin, Scott A. (2002), Sales Promotion. Cambridge, MA: Marketing Science Institute.
Orrison, A., A. Schotter, and K. Weigelt (2004), "Multiperson Tournaments: An Experimental Examination," Management Science, 50 (2), 268-79.
Reibstein, David J. and Phillis A. Traver (1982), "Factors Affecting Coupon Redemption Rates," Journal of Marketing, 46 (October), 102-113.
Shaked, Moshe and J. George Shanthikumar (1994), Stochastic Orders and Their Applications. San Diego: Academic Press.

Soman, Dilip (1998), "The Illusion of Delayed Incentives: Evaluating Future Effort-Money Transactions," Journal of Marketing Research, 35 (November), 427-37.
Taylor, Curtis R. (1995), "Digging for Golden Carrots: An Analysis of Research Tournaments," American Economic Review, 85 (4), 872-90.

Travish, Leeron and Rann Smorodinsky (2002), "Mobile Entertainment: A Generational Lifestyle Sea Change," Universal Mobile Entertainment and Cash-U white paper.
Tullock, Gordon (1980), "Efficient Rent-Seeking," in Towards a Theory of the Rent-Seeking Society, J. Buchanan, Gordon Tullock, and Robert Tollilson, eds. College Station: Texas A\&M University Press, 224-36.
Varian, Hal R. (1980), "A Model of Sales," American Economic Review, 70 (4), 651-59.
Ward, James C. and Ronald Paul Hill (1991), "Designing Effective Promotional Games: Opportunities and Problems," Journal of Advertising, 20 (3), 69-81.

Copyright of Journal of Marketing is the property of American Marketing Association and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.


[^0]:    De Liu is an assistant professor, Gatton College of Business and Economics, University of Kentucky (e-mail: de.liu@uky.edu). Xianjun Geng is an assistant professor, Department of Information Systems and Operations Management, University of Washington (e-mail: gengxj@u. washington.edu). Andrew B. Whinston is Hugh Cullen Chair Professor, Department of Information, Risk, and Operations Management, Center for Research in E-Commerce, University of Texas at Austin (e-mail: abw@ uts.cc.utexas.edu). The authors are grateful to Bin Gu, P.K. Kannan, Vijay Mahajan, Rajagopal Raghunathan, and seminar participants at the University of Texas at Austin for helpful comments and discussions. They thank the Herb Kelleher Center for Entrepreneurship for providing financial support for this research. They are also thankful for input from online entertainment executives including Todd Coleman from Wolfpack Studios, Starr Long from NCSoft, Michael Sellers from Alchemy, and Elias Slater from Gamespy.

[^1]:    ${ }^{1}$ For example (Kalra and Shi 2002), Folger's coffee advertised a contest in which consumers who could correctly identify a singer who regularly drinks Folger's would be entered into a drawing; the answer was provided in an advertisement on the same page.
    ${ }^{2}$ Federal law views promotions in which the outcome is determined by chance, in which the entry requires some form of consideration (e.g., purchase), and in which the winner is awarded a prize as lotteries. With rare exceptions, lotteries are illegal.
    ${ }^{3}$ In Europe and Japan, where this mobile game is most popular, consumers pay a per-message fee (in the SMS version) or perminute fee (in the WAP version).
    ${ }^{4}$ EAS requires contest participants to use at least one EAS product.

[^2]:    ${ }^{5}$ Other examples of consumer contests in mobile entertainment include Tetris( R ) Mobile Championship Competition and Ms. PacMan for Prizes, both of which are offered by InfoSpace Mobile, and Spot the Ball, which is offered by Planet Text.

[^3]:    ${ }^{6}$ Under the second interpretation，the skill factor is the endowed characteristic of consumers measured under the base consumption． This is because the（base）consumption may affect a consumer＇s skill factor as a result of learning effect．For example，a consumer who plays Mobile Millionaire a lot may gain a higher skill level than a consumer who plays little，even if both initially have the same skill level．

[^4]:    ${ }^{8}$ For the derivation process, see the Appendix.

[^5]:    ${ }^{9}$ We allowed this distribution density to have zero and nondifferentiable points purely for the simple representation. These zero and nondifferential points can otherwise be eliminated by redistributing the weight of distribution. None of these two features drives the results here.

[^6]:    ${ }^{10}$ Subsequently, we discuss additional strategies to increase a marketer's profits in such a setting, such as segmentation.

[^7]:    ${ }^{11}$ For a list of properties of dispersive order, see Shaked and Shanthikumar (1994).

[^8]:    ${ }^{12}$ If all consumers have the same skill level (i.e., $\underline{\mu}=\bar{\mu}$ ), not only does the equilibrium analysis herein not apply, it $\overline{\text { can }}$ also be shown that no pure-strategy equilibrium exists.
    ${ }^{13}$ In contrast, if the marketer does segmentation based on nonskill factors, such as geographic region, we call it "horizontal segmentation." From $\mathrm{P}_{6}$, we know that horizontal segmentation does not benefit the marketer.

