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Economic Analysis of Reward Advertising

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Reward advertising is an emerging monetization mechanism for app developers in which consumers choose to view ads in exchange for apps' premium content. We provide the first economic analysis of reward advertising by studying the implications of offering reward ads, either by itself, or in conjunction with direct selling of premium content. We find that the condition for offering reward ads is surprisingly simple, and it is often optimal to offer reward ads jointly with direct selling of premium content. Interestingly, a high reward rate could decrease the number of reward ads viewed because of accelerated satiation for premium content; thus, developers need to balance the need to incentivize ad viewing and to prevent excessive accelerated satiation. The need for limiting the number of reward ads per consumer only arises when the marginal revenue of reward ads diminishes quickly. Such limit is only effective when the base ad revenue rate is not too high and when ad viewers have relatively homogenous nuisance costs. Finally, reward ads may increase or decrease consumer surplus.

Key words: reward advertising; apps; monetization; premium content; consumer surplus
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1. Introduction

While the market for mobile apps is vast, there are very few choices for app developers when it comes to monetization. A predominant form of monetization is through advertising. Statista estimates that more than 90% of apps on Google Play Store (the largest app store in terms of content volume) are free and rely on advertising as one of their two main sources of revenue[1] (Statista 2018). The reliance on the advertising model creates a tension between app developers and app users: advertisements are often pushed to app users, creating nuisance costs and hindering user experiences. This tension is especially acute for mobile apps, because it is difficult for users to ignore ads on a small mobile screen and precise targeting on mobile is difficult due to limited information available about app users. As a result, mobile app developers face a difficult choice between hurting user experiences with intrusive ads and sacrificing their own profitability.

In the meantime, many app developers start to employ the "freemium" business model, where a limited version of the app is available for free, but customers can pay a price to access additional features such as premium content and functionalities (Cheng and Liu 2012, Cheng et al. 2014). However, the conversion rate from free to premium customers is usually very low, estimated to be around 2%–5% (Cohn 2015). So, it is challenging for app developers to survive on premium pricing alone.

Recently, a new monetization mechanism, called *reward advertising*, starts to gain popularity among app developers. Instead of pushing ads to consumers, reward advertising lets consumers opt in for ads in exchange for some kind of rewards. For example, in a popular collectible-card mobile game called Plants vs. Zombies Heroes, consumers are given an option of viewing a video ad to get free gems (which can then be used to exchange for gameplay-enhancing cards). In a top-selling collage-editing app called Pic Stitch, consumers can select to view video ads in exchange for

[Correction added on 14 October 2019, after first online publication: the affiliation and email address of Lin Hao has been changed from University of Notre Dame to University of Washington, to reflect the author's current affiliation.]

new collage layouts that would otherwise cost them $0.99. Reward ads are found in a broad range of apps including games, news, utilities and social networking apps, with the type of rewards ranging from virtual items, coupons, digital content, and unlocking certain app features (Heine 2013). Proponents of reward ads claim that consumers are more engaged in reward ads and more likely to develop a positive attitude about the brand (eMarketer 2014, Heine 2013). Thus, when consumers choose to view rewards ads, a win-win-win situation could arise: consumers feel compensated, app developers get ad revenue, and brands benefit from a more engaged audience (Cohen-Aslatei 2016). Such claims receive some support from a 2017 study by eMarketer that ranks reward ads the most acceptable ad format among mobile users, with 41% users finding them "acceptable."

As a nascent monetization mechanism, reward adverting has so far received very little attention in the literature, with many questions still unanswered. For example, some apps employ reward ads but many do not; it is not yet clear when reward ads should be offered. Among those employing reward ads, some offer them alone but most in conjunction with premium pricing—consumers can either directly buy premium content or functionalities, or view reward ads in exchange for them. This raises an interesting question of whether rewards ads should be offered alone or jointly. In addition, we also observe that some app developers set a maximum number of reward ads a consumer can view.[2] What motivates such limits and when should app developers set a limit on reward ads? In this study, we attempt to address a series of questions regarding optimal use of reward ads, including:

- When should reward ads be offered? If they are offered, should they be offered alone or jointly with selling premium content/features?
- How should the app developer choose the reward rate (i.e., the amount of rewards per ad viewed)?
- Should the app developer set a limit on the number of reward ads per consumer? If so, how to set such a limit?

With the above questions in mind, this study aims to provide the first economic analysis of reward advertising. We do so in a stylized context where an app developer chooses among three strategies: a pure *content-selling strategy*, where consumers can pay a unit price to obtain premium content,[3] a pure *reward–ads* strategy, and a hybrid strategy that combines the two. We ask whether and when it is profitable for an app developer to offer reward ads (alone or jointly) and if so, what is the optimal reward rate. We then study an extended model in which the app developer may set a limit for the total number of reward ads a consumer can view.

Our first key finding is that the rule for determining whether to offer reward ads is surprisingly simple: an app developer should offer reward ads if and only if the revenue generated from a reward advertisement exceeds the nuisance cost of *some* of its consumers. In other words, if the app developer can make a profit by inducing these consumers to view the reward ads, then reward advertising, either offered alone or jointly with content-selling, would be profitable.

We further find that, when reward ads are offered, it is beneficial to employ a pure reward–ads strategy if the revenue rate from reward ads is high and heterogeneity in consumers' nuisance costs from viewing the ads is low. Otherwise, the app developer should pursue a hybrid strategy, that is, offer reward ads along with premium content selling. Our further analysis shows that the hybrid strategy produces the same consumer surplus as pure content selling, but results in higher app-developer profits. In contrast, the pure reward–ads strategy may decrease or increase consumer surplus depending on the proportion of low-nuisance-cost (*low-cost* for short) consumers. In general, a pure reward-ad strategy benefits low-cost consumers and hurts high-cost ones.

We obtain insights on the optimal reward rates in different scenarios. In the hybrid strategy, the reward rate should be simply set to make low-cost consumers indifferent between viewing reward ads and buying premium content. In the pure reward–ads strategy, the reward rate should be set such that both high- and low-cost consumers prefer reward ads to direct buying. Increasing reward rates has two countervailing effects: it increases the amount of rewards per ad viewed (the "stronger incentive" effect), but may also reduce the total number of ads viewed because consumers' demand for premium content is more quickly satiated (the "accelerated satiation" effect). The optimal reward rate balances between providing high enough rate to incentivize high-cost consumers to view reward ads, and not over-incentivizing the low-cost consumers who may be at the point of excessive accelerated satiation.

Finally, we find that the need for limiting number of reward ads per consumer only arises when the revenue from each additional reward ad diminishes quickly. The diminishing marginal revenue could be a result of, for example, running out of high-quality ads to match with consumers, ad fatigue, and consumers' attempt to game the system. A limit is only effective when the base ad revenue rate is not too high (otherwise it is better to let consumers view as many as they want), and when consumers who choose to view reward ads are relatively homogenous in nuisance costs (so that the distortion caused by a uniform limit is minimal).

The rest of the study is organized as follows. We review relevant literature next, followed by modeling framework, analyses of benchmark, and main models and app developer's optimal strategy. We then extend the model to a case with reward ads limit, and conclude the study.

2. Literature Review

Our study is related to two streams of literature: revenue models for digital content and advertising. During the past decades, a vast amount of work has been dedicated to designing proper mechanisms for content providers to monetize their digital content under different market settings (Casadesus-Masanell and Zhu 2010, Cheng and Liu 2012, Fan et al. 2007, Kumar and Sethi 2009, Lee and Tan 2014, Prasad et al. 2003, Sun and Zhu 2013). Based on which revenue sources a content provider chooses to rely on, those revenue models can be divided into three categories: (i) freemium models which generate providers' revenue from selling premium content, (ii) ad-sponsored models which generate their revenue from selling ads, and (iii) a hybrid of both (i) and (ii). Cheng and Liu (2012) studied one major type of freemium models—the time-limited freemium model which provides the full functionality of software for free for a limited period—in the presence of network externalities and consumer uncertainty. Based on the trade-off between demand cannibalization and uncertainty reduction, they derived the optimal free trial time that a software vendor should offer to its customers. Cheng et al. (2014) further demonstrated differences and similarities between the time-limited freemium model, and another major type of freemium model called the feature-limited model which provides the basic functionality of software for free for an unlimited period while charging a positive price for the remaining premium functionality. They have demonstrated that the intensity of the network effect is a major factor that decides which model (time-limited, feature-limited, and hybrid) generates the largest profit for a provider. Lee and Tan (2014) empirically identified factors that drive greater adoption for the time-limited and the feature-limited freemium model. They found no significant differences between the two types of freemium models regarding product sampling performance.

Regarding ad-sponsored models and hybrid models, the extant literature highlights two central issues: (i) when a content provider should adopt an ad-sponsored or a hybrid model, and (ii) the impact of such adoption on market outcomes like content prices and providers' profit. Concerning the latter issue, Gabszewicz et al. (2005) showed that a monopolistic media firm would lower its newspapers' price when it starts to profit from advertising spaces. Hao et al. (2017) showed that a mobile app developer would reduce the app price to enlarge its app user base if she chooses to become an in-app ad publisher. Sun and Zhu (2013) empirically found that content providers would tailor their content as well as improve their content quality once they start to earn ad profit from the traffic to their content. Concerning the question when a provider should adopt an ad-sponsored or a hybrid model, Fan et al. (2007) showed that a media company should focus on selling its content only, when online access cost is low and content quality is relative high. When online access cost is high, the firm should start to sell advertising. Kumar and Sethi (2009) investigated the scenario where a provider under can dynamically adjust both its content price and ad level over time. Based on the control theory, they derived the optimal content price and the optimal ad level as functions of time. Prasad et al. (2003) investigated a scenario where the provider offers consumers two versions of content—one with a low content price but a high level of advertising and the other with a high content price but a low level of advertising. Consumers will self-select to either choice based on their valuation for the content and their nuisance cost, leading to either a pooling or a separating outcome in equilibrium. The authors specifically demonstrated when the firm should induce a pooling or a separating outcome.

Our study is also related to the literature on advertising (Anderson and Coate 2005, Anderson and Gabszewicz 2006, Bagwell 2007, Liu et al. 2012). For online advertising, Asdemir et al. (2012) investigated advertisers' optimal choices between two popular advertising models—cost per impression (i.e., CPM model) and cost per click (i.e., CPC model). They identified that four interrelated factors that drive the advertisers' optimal choices—uncertainty over the boundaries of the target segment, the advertising value of the target segment, the cost of mistargeting ads to the nontarget segment, and the discrepancy in the alignment of incentives of the advertiser and the publisher. Liu and Viswanathan (2012) investigated publishers' optimal choices among three types of advertising models, pay-for-impression (e.g., CPM model), pay-for-performance (e.g., CPC model) and the hybrid model in the presence of a two-sided information asymmetry where both publishers and advertisers are uncertain about the other parties' quality. Dellarocas (2012) demonstrated that one type of pay-for-performance model (i.e., the pay-for-action model) might distort product prices upwardly and consequently leads to both inferior advertiser-publisher profit and inferior consumer surplus if product prices are endogenized to reflect advertisers' ad expense. Athey and Gans (2010) developed a theoretical model to investigate the impact of ad targeting on advertisers' participation decisions. They showed that ad prices could decrease in targeting performance as better targeting increases the total number of advertisers who can be accommodated.

Chen and Stallaert (2014) focused on the impact of behavioral targeting on the publishers' side. They demonstrated that better targeting performance does not always benefit a publisher and identified the market conditions when it does.

A key difference between reward advertising and traditional ad-sponsored models lies in who decides the advertising level. The ad-sponsored revenue model focuses on the scenario where the content provider decides the advertising level—advertisement is "pushed" to content consumers who have no control over its intensity. We study a new scenario where consumers choose the number of reward ads to view —advertisement is "pulled" by content consumers who can opt for different levels of ads. This shift of control from the content provider to consumers can alter advertising efficiency, surplus distribution, and consumer incentives. Meanwhile, traditional ad-sponsored models have offered premium content either (i) independent of ad consumption or (ii) in conjunction with ad removal. In reward advertising, access to premium content is tied to increased ad consumption. Such a different arrangement suggests that reward ads will be viewed differently, and insights from traditional ad-sponsored models may not be applicable to reward ads. That said, we consider our research on this novel form of advertising complements existing research on traditional advertising models, for the two types of advertising can be combined.

3. Modeling Framework

We consider a monopoly app developer offering premium content to a unit mass of consumers who have already adopted the app.[4] Consumers are heterogeneous in terms of their valuation for the premium content. We assume that consumers' valuation v follows a uniform distribution on $[0, V]$, where V represents the highest consumer valuation. Each consumer may acquire multiple units of premium content. For example, in Plants vs. Zombies Heroes, players might be interested in acquiring multiple gameplay-enhancing cards. By acquiring (and consuming) x units of premium content, consumers with valuation v gains a gross utility of $vx - \frac{x^2}{2}$. The quadratic term captures the diminishing marginal return of premium content, meaning that each additional unit of the premium content brings less utility than the unit before. Most consumption goods have diminishing marginal returns (Abhishek et al. 2016, Chellappa and Mehra 2018, Sundararajan 2004).[5]

In addition to their valuation for the premium content, consumers are also heterogeneous in nuisance cost from viewing a reward ad, denoted by c. We assume c follows a discrete distribution: a λ $(0 < \lambda < 1)$ proportion of consumers are *low-cost* consumers with a low nuisance cost of c_L, while the rest $(1-\lambda)$ proportion of consumers are *high-cost* ones with a high nuisance cost of c_H $(c_L < c_H)$. In summary, consumers are heterogeneous in both valuation v and nuisance cost c and are distributed on Uniform$[0, V] \times \{c_L, c_H\}$.

The app developer may offer different ways for consumers to acquire premium content: buying premium content or viewing reward ads in exchange for premium content. We assume the buying option is always available and consider the following two models: (i) a benchmark model without the option of viewing reward ads; (ii) a main model with the option of viewing reward ads.

In the benchmark model, consumers can acquire the premium content only through buying. Suppose a consumer with valuation v acquires $x = q$ units of premium content through buying. Then his net utility u is $u = vq - \frac{q^2}{2} - pq$.

Each consumer chooses his buying quantity $q(v)$ to maximize his net utility u. We note that in the benchmark model a consumer's choice of buying quantity $q(v)$ depends on his valuation v for the premium content, but not on his nuisance cost c. Consumers with $v > p$ will buy a positive quantity of premium content. We use Q to denote the total units of premium content sold across all consumers. We have

$$Q = \int_p^V \frac{q}{V} dv.$$

Hence, the app developer's profit π is

$$\pi = pQ.$$

The app developer chooses unit price p to maximize her profit π.

We now turn to the main model, where consumers can acquire premium content either through buying or viewing reward ads. Suppose for each ad viewed, a consumer gets r units of premium content. We refer to r as the *reward rate* for viewing ads, which is determined by the app developer. We use n to denote the number of ads viewed by a consumer. Then the number of premium content acquired through viewing ads is rn and the corresponding nuisance cost is cn (Anderson and Coate 2005, Anderson and Gabszewicz 2006, Hao et al. 2017), where $c = c_L$ or c_H. Suppose the consumer also buys q units of premium content. Thus, the total quantity of premium content acquired is $q + rn$, and the consumer (v, c)'s net utility u is

$$u = v(q + rn) - \frac{(q + rn)^2}{2} - pq - cn.$$

Each consumer chooses his buying quantity q and ad viewing quantity n to maximize his net utility u. We note that in the main model a consumer's buying quantity $q(v,c)$ and ad viewing quantity $n(v,c)$ depend both on his valuation v and on his nuisance cost c. We have the following Lemma 1. The proofs of all lemmas and propositions are relegated to the online Appendix S1.

LEMMA 1. (BUYING QUANTITY VS. VIEWING QUANTITY). *The choice between buying premium content and viewing reward ads in exchange for premium content depends on their relative costs. If $p < \frac{c}{r}$, a consumer prefers buying to viewing ads. Otherwise, he prefers viewing ads to buying.*

Intuitively, Lemma 1 states that to obtain one unit of content, a consumer either pays price p or nuisance cost $\frac{c}{r}$, whichever is lower. Consumers do not mix the two approaches because the "cheaper" approach is always cheaper due to constant marginal costs of both options.

Suppose among the low (high) nuisance-cost consumers, the one with $\hat{v}_L$ ($\hat{v}_H$) is indifferent between acquiring premium content (via buying or viewing reward ads) and not. Consumers with valuation greater than $\hat{v}_L$ ($\hat{v}_H$) in the low (high) nuisance-cost group will acquire a positive number of premium content. Then, the total number of premium content sold across all consumers (Q) is

$$Q = \lambda \int_{\hat{v}_L}^{V} \frac{q}{V} dv + (1-\lambda) \int_{\hat{v}_H}^{V} \frac{q}{V} dv.$$

Denote N as the total number of ads viewed across all consumers, and we have $N = \lambda \int_{\hat{v}_L}^{V} \frac{q}{V} dv + (1-\lambda) \int_{\hat{v}_H}^{V} \frac{q}{V} dv$. We use a to denote the *revenue rate* for advertising, that is, how much advertising revenue the app developer can generate through one unit of ad view. Hence, the app developer's profit π is

$$\pi = pQ + aN.$$

The app developer chooses price p and reward rate r to maximize her profit π.

The sequence of decisions is as follows: In the benchmark model where the app developer offers the buying option only, the app developer chooses unit price p in Stage 1 and consumers choose their buying quantity q in Stage 2. In the main model where the app developer offers the buying option and reward ads, the app developer chooses unit price p and reward rate r in Stage 1 and consumers choose their buying quantity q and ad viewing quantity n in Stage 2. The following is a summary of notation (Table 1).

4. Analysis of the Benchmark Model: No Reward Ads

Under the benchmark model, the app developer offers the buying option only and consumers may pay price p for a unit of the premium content. Each consumer decides purchase quantity q to maximize his net utility $u = vq - \frac{q^2}{2} - pq$. Therefore, the optimal purchase quantity is $v - p$ for consumers with $v > p$, while the optimal purchase quantity is zero for consumers with $v \leq p$. Thus, the total number of premium content sold $Q = \int_{p}^{V} (v-p)\frac{1}{V} dv$. Then the app developer decides selling price p to maximize her profit $\pi = pQ$. Solving the app developer's profit optimization problem yields Lemma 2.

LEMMA 2. (PURE CONTENT-SELLING STRATEGY). *When the app developer only sells premium content, she optimally sets $p = \frac{V}{3}$. The corresponding profit is $\pi = \frac{2V^2}{27}$.*

Table 1 Summary of Notation

App developer's decision variables	
p	Price for a unit of premium content
r	Reward rate for viewing an ad, measured in units of premium content
$\bar{n}$	Maximum number of ads a consumer can view, a decision variable for the app developer in the extended model
Consumers' decision variables	
q	Number of premium content bought by an individual consumer
n	Number of reward ads viewed by an individual consumer
Other variables	
π	App developer's profit
u	Net utility for a consumer
x	Number of premium content acquired/consumed by an individual consumer (through either buying or viewing reward ads)
X	Number of premium content acquired/consumed by all consumers
Q	Total number of premium content sold to all consumers
N	Total number of reward ads viewed by all consumers
m	Number of premium consumers who acquire premium content (through buying or viewing reward ads)
CW	Consumer surplus
Parameters	
v	Consumers' valuation of premium content and $v \sim \text{Uniform}[0, V]$
V	Highest consumer valuation
c_L and c_H	Consumers' nuisance cost per ad view
λ	Proportion of low-cost consumers and $0 < \lambda < 1$
a	Revenue rate for advertising

Consequently, consumers with $v \in \left[\frac{V}{3}, V\right]$ will buy premium content and consumers with $v \in \left[0, \frac{V}{3}\right)$ will not.

The intuition behind this lemma is simple. When the app developer offers the buying option only, consumers buy premium content if and only if their valuations are higher than the price. A lower price encourages consumers to buy more premium content. Therefore, the app developer chooses the price to optimally trade off the profit margin and the sales volume, which leads to two-thirds of the market being covered.

5. Analysis of the Main Model: With the Option of Reward Ads

Under the main model, the app developer offers consumers two ways to acquire premium content: buying and viewing reward ads. The app developer sets the price for premium content p and reward rate r; consumers choose between buying premium content, viewing reward ads, or neither. The cost of buying one unit of premium content is its unit price p, whereas the cost of acquiring one unit of premium content through viewing reward ads is $\frac{c}{r}$, where $c = c_L$ or c_H. Naturally, consumers who acquire premium content would compare the acquisition costs and choose the option with the least cost. As such, no one would choose both options at the same time. Therefore, there are three scenarios depending on how p compares to $\frac{c_L}{r}$ and $\frac{c_H}{r}$. When the app developer sets a low price for premium content such that $p < \frac{c_L}{r}$, some consumers buy premium content and no one chooses viewing ads. We call the decisions leading to this scenario the *pure content-selling strategy*. When the app developer sets a medium price for premium content such that $\frac{c_L}{r} \leq p < \frac{c_H}{r}$, low-cost consumers prefer viewing ads while high-cost consumers prefer buying premium content. We call the decisions leading to this scenario the hybrid strategy. When the app developer sets a high price for premium content such that $p \geq \frac{c_H}{r}$, both low- and high-cost consumers prefer viewing ads over buying premium content. We call the decisions leading to this scenario pure reward-advertising strategy. Note that under the pure content-selling strategy, the problem facing the app developer is identical to that in the benchmark model, we thus refer to both scenarios as the pure content-selling strategy. Figure 1 illustrates the consumer decisions under the three strategies.

Because the app developer and consumers' decisions under the pure content-selling strategy have already been analyzed in the benchmark model, we next focus on analyzing their decisions under the other two strategies, hybrid and pure reward-advertising.

5.1. Hybrid Strategy

Under the hybrid strategy, the app developer sets $\frac{c_L}{r} \leq p < \frac{c_H}{r}$, and consequently low-cost consumers prefer viewing ads while high-cost consumers prefer buying premium content. A high-cost consumer decides purchase quantity q to maximize his net utility $u = vq - \frac{q^2}{2} - pq$. A low-cost consumer determines number of ads viewed n to maximize his net utility $u = v(rn) - \frac{(rn)^2}{2} - cn$. For high-cost consumers, the optimal purchase quantity is $v - p$ for consumers with $v > p$, while the optimal purchase quantity is zero for high-cost consumers with $v \leq p$. For low-cost consumers, the optimal number of ads viewed is $n = \frac{rv - c_L}{r^2}$ for low-cost consumers when $c_L < rv$ and $n = 0$ for low-cost consumers when $c_L \geq rv$.

Given the consumer decision-making process, we characterize the app developer's decision making. The total number of reward ads viewed by low-cost customers $N = \lambda \int_{c_L/r}^{V} \left(\frac{rv - c_L}{r^2 V}\right) n dv$ and total number of premium content sold $Q = (1 - \lambda) \int_{p}^{V} \left(\frac{v - p}{V}\right) dv$. The app developer decides p and r simultaneously to maximize profit $\pi = pQ + aN$. Lemma 3 presents the app developer's optimal price p and reward rate r under the hybrid strategy.

Lemma 3. (Hybrid Strategy). *Under the hybrid strategy, the app developer optimally sets $p = \frac{V}{3}$ and $r = \frac{3c_L}{V}$. The corresponding profit is $\pi = \frac{2V^2(c_L - c_L\lambda + a\lambda)}{27c_L}$. Consequently, low-cost consumers with $v \in \left[\frac{V}{3}, V\right]$ view ads but do not buy; high-cost consumers with $v \in \left[\frac{V}{3}, V\right]$ buy premium content but do not view ads; and other consumers neither view nor buy.*

Under the hybrid strategy, consumers with different nuisance costs self-select into different options of acquiring premium content: low-cost consumers choose viewing ads while high-cost consumers choose buying premium content. As in Lemma 1, the optimal price in the pure content-selling case is still optimal in the hybrid case, which leads to the same revenue from high-cost consumers. However, the developer earns a different amount of revenue from low-cost consumers compared to the pure content-selling case because the low-cost consumers now choose to view reward ads to acquire the premium content. The addition of reward ads provides the app developer an opportunity to discriminate consumers based on their nuisance costs, which does not exist under the pure content-selling strategy.

5.2. Pure Reward-Advertising Strategy

Under the pure reward-advertising strategy, the app developer sets $p \geq \frac{c_H}{r}$, and consequently both low- and high-cost consumers prefer viewing ads over buying premium content. A consumer decides number of ads viewed n to maximize his net utility $u = v(rn) - \frac{(rn)^2}{2} - cn$. Therefore, the optimal number of ads viewed $n = \frac{rv - c_L}{r^2}$ for low-cost consumers and $n = \frac{rv - c_H}{r^2}$ for high-cost consumers. Thus, the total number of reward ads viewed $N = \lambda \int_{c_L/r}^{V} \left(\frac{rv - c_L}{r^2 V}\right) dv + (1 - \lambda) \int_{c_H/r}^{V} \left(\frac{rv - c_H}{r^2 V}\right) dv$ and number of premium content sold Q = 0. The app developer determines reward rate r to maximize profit $\pi = pQ + aN$. Lemma 4 provides the app developer's optimal reward rate under this strategy.

Lemma 4. (Pure Reward-Advertising Strategy). *Under the pure reward–advertising strategy,*

(*a*) *both-view-ads: If* $\lambda \leq \frac{1}{4}$, *or* $\frac{1}{4} < \lambda < \frac{3}{4}$ & $\frac{c_H}{c_L} \leq \frac{4\lambda(1-\lambda) + \sqrt{3\lambda(1-\lambda)}}{(1-\lambda)(4\lambda - 1)}$, *or* $\lambda \geq \frac{3}{4}$ & $\frac{c_H}{c_L} \leq 3$, *then the app developer optimally sets* $r = \frac{2\lambda c_L + 2(1-\lambda)c_H + \sqrt{\Delta}}{V}$ (*the definition of* Δ *can be found in the proof of Lemma* 4 *in online Appendix S*1) *and p takes any value satisfying* $p \geq \frac{c_H}{r}$. *The corresponding profit is* $\pi = \frac{aV^2\left(\left(c_H + \sqrt{\Delta}\right)^2 - \lambda(c_H - c_L)\left(c_L + c_H + 2\sqrt{\Delta}\right)\right)}{2\left(2\lambda c_L + 2(1-\lambda)c_H + \sqrt{\Delta}\right)^3}$. *Consequently, low-cost consumers with* $v \in \left[\frac{c_L V}{2\lambda c_L + 2(1-\lambda)c_H + \sqrt{\Delta}}, V\right]$ *and high-cost consumers with* $v \in \left[\frac{c_H V}{2\lambda c_L + 2(1-\lambda)c_H + \sqrt{\Delta}}, V\right]$ *view ads but do not buy; and other consumers neither view nor buy.*

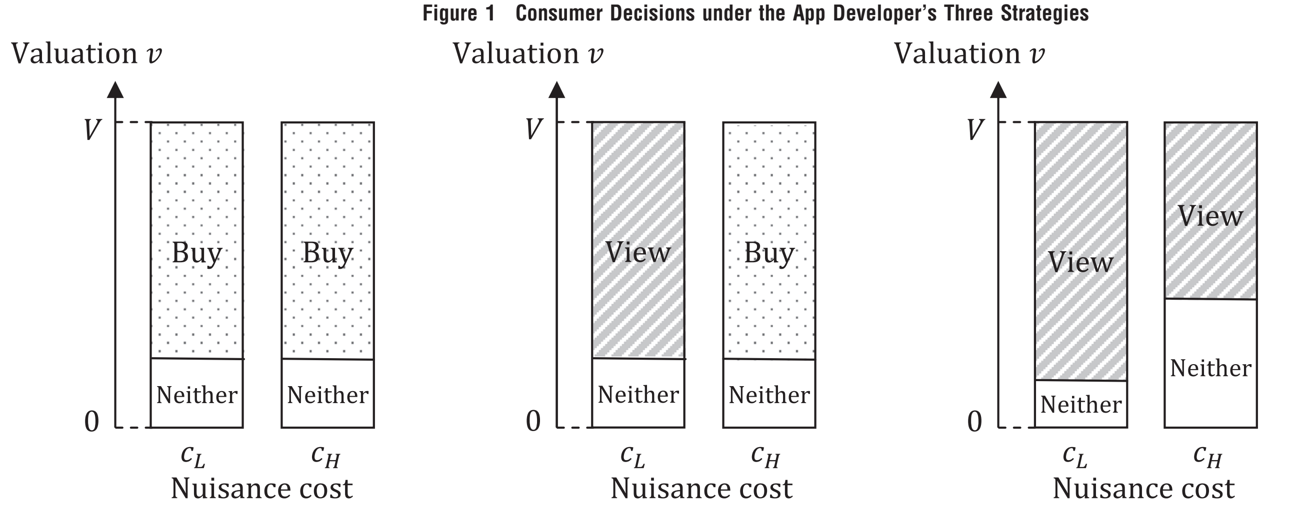
(*b*) *only-low-type-view-ads: Otherwise, the app developer optimally sets* $r = \frac{c_H}{V}$ *and p takes any value satisfying* $p \geq \frac{c_H}{r}$. *The corresponding profit is* $\pi = \frac{aV^2(c_H - c_L)^2 \lambda}{2c_H^3}$. *Consequently, low-cost consumers with* $v \in \left[\frac{c_L V}{c_H}, V\right]$ *view ads but do not buy; and other consumers neither view nor buy.*

Under the pure reward–advertising strategy, because the app developer's revenue is proportional to the total number of ads viewed, she optimally sets reward rate r to maximize total number of ads viewed N. Lemma 4 and Figure 2 show that the app developer may serve either both low- and high-cost consumers, or only low-cost consumers under the pure reward-advertising strategy. Under the pure reward-advertising strategy, low-cost consumers are more valuable because they tend to view more ads and contribute more to N than high-cost consumers. Therefore, if the proportion of low-cost consumers is low (i.e., λ is small), or if consumer heterogeneity in nuisance cost is low (i.e., $\frac{c_H}{c_L}$ is small), then the app developer serves both low- and high-cost consumers. Otherwise, the app developer only serves low-cost consumers.

6. App Developer's Optimal Strategies and Their Properties

In this section, we first examine how the app developer should choose between the pure content-selling, hybrid, and pure reward-advertising strategies under

Figure 1 Consumer Decisions under the App Developer's Three Strategies



(a) Pure content-selling **(b)** Hybrid **(c)** Pure reward-advertising

Notes: **View = View reward ads, Buy = Buy premium content, and Neither = Neither view reward ads nor buy premium content.**

Figure 2 App Developer's Optimal Decisions under the Pure Reward-Advertising Strategy

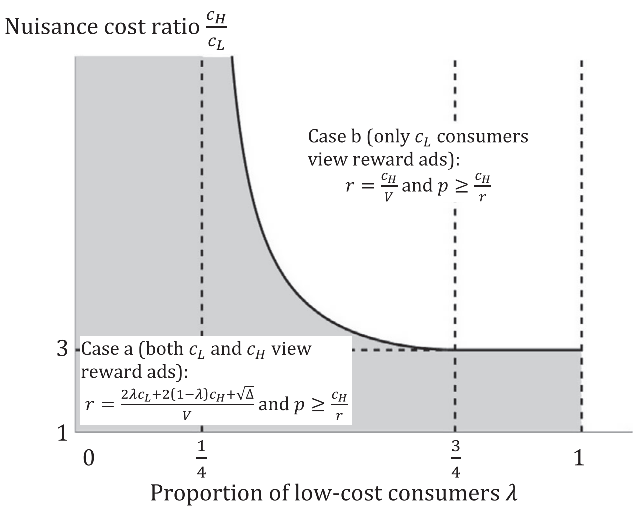
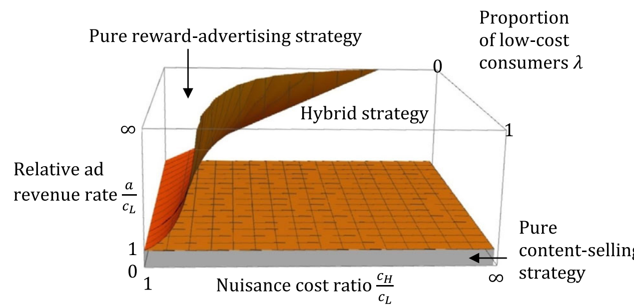


Figure 3 App Developer's Optimal Strategies [Color figure can be viewed at wileyonlinelibrary.com]



various market conditions. We then examine characteristics of the optimal reward rate under the hybrid and pure reward-advertising strategies. Finally, we explore the impact of the app developer's strategy on the number of premium consumers (i.e., who acquire premium content through buying or viewing ads), total premium content consumption, and consumer surplus.

6.1. App Developer's Optimal Strategies

The app developer compares the profits under the three strategies and chooses one to maximize her profit. We use superscripts *CS*, *Hybrid*, and *RA* to represent the pure content-selling, hybrid, and pure reward-advertising strategies, respectively. Proposition 1 summarizes the app developer's optimal strategies for various market conditions.

Proposition 1. (App Developer's Optimal Strategies). *Comparing the profits under three strategies yields:*

(*a*) *If* $\frac{a}{c_L} \leq 1$, *then the pure content-selling strategy is optimal.*
(*b*) *If* $\frac{a}{c_L} > \hat{a}$ *&* $\lambda < \frac{3}{4}$ *&* $\frac{c_H}{c_L} < \frac{(1-\lambda)(1+2\lambda)+\sqrt{1-\lambda}}{2\lambda(1-\lambda)}$, *or* $a > \hat{a}$ *&* $\lambda \geq \frac{3}{4}$ *&* $\frac{c_H}{c_L} \leq 3$, *then the pure reward-advertising strategy case a* (*both-view-ads*) *is optimal*
(*c*) *Otherwise, the hybrid strategy is optimal. The definition of can be found in the proof of Proposition* 1 *in online Appendix S*1.

As shown in Proposition 1 and illustrated in Figure 3, when the ad revenue rate is low relative to consumers' nuisance cost (i.e., $\frac{a}{c_L}$ is low), offering reward ads is not as profitable as directly selling premium content. Thus, the app developer should adopt the pure content-selling strategy. When the ad revenue rate is high relative to consumers' nuisance cost (i.e., high $\frac{a}{c_L}$), it is profitable for the app developer to offer reward ads alone (the pure reward-advertising strategy) or in combination with content selling (the hybrid strategy). When reward ads are offered, the pure reward-advertising strategy is more profitable if consumers are highly homogeneous in nuisance costs (i.e., low nuisance cost ratio $\frac{c_H}{c_L}$); otherwise, the hybrid strategy is more profitable. This is because the hybrid strategy provides the app developer an opportunity to discriminate consumers based on their nuisance costs, whereas the pure reward-advertising strategy tries to serve all consumers reward ads, which is not ideal when consumers have very different nuisance costs and thus require very different optimal reward rates.

Note that the pure reward-advertising strategy case *b* (Lemma 4), that is, serving only low-cost consumers, is always dominated by the hybrid strategy. This is because, under the pure reward-advertising strategy, when the consumer heterogeneity in the nuisance cost is high and the proportion of low cost consumers is also high, the app developer should serve low-cost consumers only. Serving high-cost consumers in such a case would require so high a reward rate that hurts her revenue from low-cost consumers. Rather than serving low-cost consumers alone, the app developer would be better off by adopting the hybrid strategy because she can sell premium contents to high-cost consumers, without hurting her advertising revenue from low-cost consumers.

6.2. Characteristics of Reward Rate r

Reward rate r is an important decision for the app developer in reward advertising. To gain more insights on this decision variable, we explore how the optimal reward rate r changes with model parameters and between app developer strategies. Prior to our analysis, it is useful to understand the effect of reward rate r on consumers' ad viewing quantity n. As presented in Lemma 5, a higher reward rate does not necessarily lead to a higher number of ad views for a consumer.

LEMMA 5. (IMPACT OF REWARD RATE r ON CONSUMERS' AD VIEWING QUANTITY n). *In both the pure reward-advertising strategy and the hybrid strategy, when a consumer's nuisance cost $c < \frac{rv}{2}$ (where $c = c_H$ or c_L), the consumer's ad viewing quantity n decreases in r; otherwise, n increases in r.*

When reward rate r increases, there are two effects. On the one hand, the benefit from viewing an ad increases because more units of premium content are rewarded per ad viewed. We refer to this effect as the *stronger-incentive effect*. On the other hand, as consumers obtain more premium content, the marginal utility of premium content decreases. Increasing reward rate r accelerates this process, thus may decrease the number of ads viewed, leading to an *accelerated-satiation effect*. For example, consider a consumer with $v = 5$ and $c = 1$. A reward rate of 1 unit of content per ad will lead the consumer to consume 4 ads (or 4 units of content). However, noting that the consumer will never consume more than 5 units of content, a reward rate of 5 will lead the consumer to consume 1 ad (or 5 units of content). The accelerated-satiation effect is more pronounced when the nuisance cost is low because a low-cost consumer is more likely to consume a large number of ads and reach her point of satiation.

The above two effects of reward rate r jointly determine the results in Lemma 5. The stronger-incentive effect encourages consumers to view more ads, whereas the accelerated-satiation effect discourages consumers from viewing ads. When nuisance cost c is low, a consumer views many ads and, thus, the accelerated-satiation effect is pronounced and dominates the stronger-incentive effect. Therefore, in such a case, number of ads viewed n decreases in reward rate r. In contrast, when nuisance cost c is high, a consumer views few ads and, thus, the accelerated-satiation effect is small and dominated by the stronger-incentive effect. Therefore, in such a case, number of ads viewed n increases in reward rate r.

Next, we examine the impact of market conditions on the optimal reward rate under the pure reward–advertising strategy. Under this strategy, the app developer's optimal reward rate r^{RA} critically depends on two market conditions: proportion of low-cost consumers λ and nuisance cost ratio $\frac{c_H}{c_L}$. Nuisance cost ratio $\frac{c_H}{c_L}$ measures the degree of consumer heterogeneity. A higher $\frac{c_H}{c_L}$ indicates that consumers are more heterogeneous in nuisance costs. The effects of these two market conditions, together with comparison of optimal reward rates under the pure reward-advertising and hybrid strategies, are summarized in Proposition 2.

PROPOSITION 2. (CHARACTERISTICS OF THE OPTIMAL REWARD RATE). *The optimal reward rate for the pure reward-advertising strategy has the following properties:*

(*a*) *r^{RA} always decreases in λ.*
(*b*) *When $\lambda \geq \frac{1}{2}$ & $\frac{c_H}{c_L} \geq \frac{2\sqrt{\lambda}}{2\sqrt{\lambda}-1}$, r^{RA} decreases in $\frac{c_H}{c_L}$; otherwise, r^{RA} increases in $\frac{c_H}{c_L}$.*
(*c*) *r^{RA} is always higher than the optimal reward rate for the hybrid strategy, that is, $r^{RA} \geq r^{Hybrid}$.*

The intuition of Proposition 2 is as follows. When the app developer decides the reward rate for the pure reward–advertising strategy, she maximizes her profit by maximizing the total number of ads viewed. According to Lemma 5, the number of ad views decreases in reward rate r when a consumer's nuisance cost is low and increases in r when that is high. Thus, to improve ad views from low-cost consumers, the app developer should lower the reward rate. In contrast, to improve ad views from high-cost consumers, the app developer should increase the reward rate. Therefore, when the proportion of low-cost consumers on the market (λ) increases, the app developer focuses more on low-cost consumers and, thus, the optimal reward rate decreases, that is, r^{RA} always decreases in λ.

Similarly, when both proportion of low-cost consumers λ and nuisance cost ratio $\frac{c_H}{c_L}$ are high, low-cost consumers are far more valuable to the app developer than high-cost consumers. Consequently, the app developer focuses more on low-cost consumers and, thus, reduce the reward rate. Therefore, reward rate r^{RA} decreases as $\frac{c_H}{c_L}$ increases. Otherwise, the app developer puts similar weights on high- and low-cost consumers. To motivate high-cost consumers to view the ads, the app developer increases reward rate r^{RA} as $\frac{c_H}{c_L}$ increases.

The intuition for the third result in Proposition 2 is as follows. Under the hybrid strategy, only low-cost consumers view ads. Under the pure reward-ad strategy, however, both low- and high-cost consumers view ads. To induce high-cost consumers to view ads, the app developer needs to raise the reward rate. Therefore, the optimal reward rate for the pure reward-ad strategy is always higher than that for the hybrid strategy.

6.3. Comparing the Three Strategies on Number of Premium Consumers, Total Premium Content Consumption, and Consumer Surplus

In this subsection, we compare the pure content-selling, hybrid, and pure reward-advertising strategies on three important economic outcomes: number of premium consumers, total consumption of premium content, and consumer surplus. We use number of premium consumers to refer to the number of consumers who acquire premium content through buying or viewing reward ads. We use m^{RA}, m^{Hybrid}, m^{CS} to denote the number of premium consumers under the pure reward-advertising, hybrid, and pure content-selling strategies, respectively.

Proposition 3. (Number of Premium Consumers and Premium Content Consumption). *The comparison among the strategies in terms of number of premium consumers and premium content consumption shows the following:*

(*a*) *The number of premium consumers under the hybrid strategy is the same as the pure content-selling strategy, and both are lower than the pure reward-ad strategy, that is,* $m^{CS} = m^{Hybrid} \leq m^{RA}$.
(*b*) *The premium content consumption under the hybrid strategy is the same as the pure content-selling strategy, and both are higher than the pure reward-ad strategy, that is,* $X^{CS} = X^{Hybrid} \geq X^{RA}$.

Proposition 3 offers some additional information on the number of premium consumers and premium content consumption. Such information is useful for certain app developers who are not only concerned with profit maximization, but also with share of paying users or quantity of premium content needs to be offered. For example, an app developer who is at the market expansion stage may favor the pure reward ad strategy; an app developer who has limited premium content inventory may be better off with a hybrid or pure content selling strategy.

Finally, we explore how the app developer's strategies affect total consumer surplus. Total consumer surplus, denoted by CW, is the summation of the net utilities of all consumers. In our model, a consumer who has no access to premium content gets zero net utility. Thus, we only need to consider consumers who has access to premium content (by buying it or by viewing reward ads). Specifically, consumer surplus under the three strategies are:

$$CW^{CS} = \int_p^V \frac{u(v)}{V} dv$$

$$\begin{aligned} CW^{Hybrid} &= \lambda CW_L^{Hybrid} + (1-\lambda) CW_H^{Hybrid} \\ &= \lambda \int_{c_L/r}^{V} \frac{u(v, c_L)}{V} dv + (1-\lambda) \int_p^V \frac{u(v, c_H)}{V} dv \end{aligned}$$

$$\begin{aligned} CW^{RA} &= \lambda CW_L^{RA} + (1-\lambda) CW_H^{RA} \\ &= \lambda \int_{c_L/r}^{V} \frac{u(v, c_L)}{V} dv + (1-\lambda) \int_{c_H/r}^{V} \frac{u(v, c_H)}{V} dv \end{aligned}$$

Proposition 4 compares these consumer surplus.

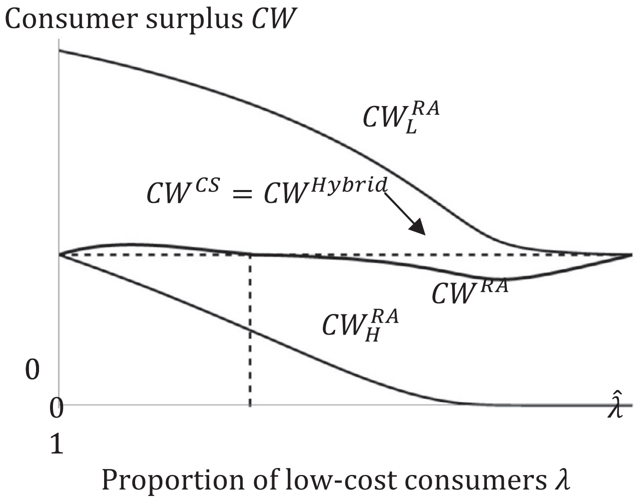
Proposition 4. (Comparison of Consumer Surplus). *Consumer surplus under the hybrid strategy is the same as that under the pure content-selling strategy.*[6] *Both are higher than the consumer surplus under the pure reward–advertising strategy* ($CW^{CS} = CW^{Hybrid} \geq CW^{RA}$), *when proportion of low-cost consumers* λ *is higher than a threshold* $\hat{\lambda}$ *(see proof of Proposition 4 in online Appendix S1 for the definition), and are lower* ($CW^{CS} = CW^{Hybrid} < CW^{RA}$), *otherwise. Furthermore,* $CW^{CS} = CW_L^{Hybrid} < CW_L^{RA}$ *and* $CW^{CS} = CW_H^{Hybrid} > CW_H^{RA}$.

Figure 4 illustrates the consumer surpluses under different strategies. In Figure 4, the dashed horizontal line represents the overall consumer surplus under the hybrid and pure content-selling strategies, which are identical, and the thick solid curve in the middle represents the overall consumer surplus under the pure reward-advertising strategy. Under the hybrid and pure content-selling strategies, low- and high-cost consumers have the same consumer surpluses as indicated by the dashed line. Low-cost consumers get a higher surplus under the pure reward-advertising strategy, while high-cost consumers obtain a lower surplus. As shown in Figure 4, the overall consumer surplus under the pure reward–advertising strategy (CW^{RA}) is higher than under two other strategies ($CW^{CS} = CW^{Hybrid}$) when λ is smaller than a threshold $\hat{\lambda}$, and lower when λ is greater than $\hat{\lambda}$.

Proposition 4 and Figure 4 show that low-cost consumers favor the pure reward–advertising strategy while high-cost consumers benefit more from the pure content-selling and the hybrid strategies. This is because the reward rate is higher under the pure reward–advertising strategy, while the selling price is lower under the hybrid and the pure content-selling strategies. Therefore, low-cost consumers have a higher surplus under the pure reward–advertising strategy while high-cost consumers have a higher surplus under the hybrid and the pure content-selling strategies.

Since low-cost consumers benefit from pure reward–advertising strategy while high-cost consumers benefit from the other two strategies, one may expect that the pure reward-advertising strategy should provide a higher overall consumer surplus than the other two strategies when the proportion of low-cost consumers is high. However, Proposition 4 and Figure 4 show an opposite result. This is because the reward rate under the pure reward–advertising strategy r^{RA} decreases in the proportion of low-cost consumers (Proposition 2). Thus, the surplus gained by each low-cost consumer (CW_L^{RA}) decreases in the proportion of low-cost consumers (λ), as shown by the top decreasing solid curve in Figure 4. As a result, although there are more

Figure 4 Comparison of Consumer Surplus



low-cost consumers as λ increases, the total consumer surplus (CW^{RA}) may decrease. This leads to the counterintuitive result that the pure reward–advertising strategy yields a higher (lower) overall consumer surplus than the other two strategies when the proportion of low-cost consumers is low (high).

7. Extended Model

In this section, we analyze an extended model in which the app developer may set the maximum number of ads a consumer can view, denoted by $\bar{n}$. With the limit in place, once a consumer's ad viewing quantity reaches $\bar{n}$, the only option for the consumer to acquire more premium content is through buying.

Intuitively, with a constant ad revenue rate, the app developer has no reason to limit the number of reward ads for each consumer. Therefore, we generalize the main model by allowing a diminishing ad revenue rate. Specifically, for a consumer who views n reward ads, the corresponding generated ad revenue is as follows:

$$\int_0^n \max\{a-bt,0\}dt = \begin{cases} an - \frac{bn^2}{2}, & \text{if } n \le \frac{a}{b} \\ \frac{a^2}{2b} & \text{otherwise} \end{cases},$$

where a represents the *base ad revenue rate* and b denotes the *diminishing speed* of unit ad revenue.

The diminishing ad revenue assumption is motivated by a few observations. First, the matching quality may decline with the number of ads served. The app developer generally should serve the best-matching ads to the customer, but because the ad inventory is limited, the high-quality matches can run out, which leads to a diminishing ad revenue rate. Second, consumers may experience "ad fatigue"; the more ads a consumer views, the more he learns to tune out on ads, causing the ad revenue to decline. Finally, consumers who choose to view many ads are more likely to game the system (e.g., by using a bot or looking elsewhere when the ad is shown), which can also lead to a declining ad revenue rate. Our assumption of diminishing ad revenue rate is consistent with earlier work (Kumar et al. 2007, Li et al. 2013).

Base on the diminishing ad revenue rate assumption, the app developer's total ad revenue is

$$\lambda \int_{\hat{v}_L}^{V} \frac{\int_0^n \max\{a-bt,0\}dt}{V} dv + (1-\lambda) \int_{\hat{v}_H}^{V} \frac{\int_0^n \max\{a-bt,0\}dt}{V} dv.$$

The formula for the app developer's total revenue from selling premium content remains the same as in the main model. The app developer chooses unit price p, reward rate r, and maximum ad viewing quantity $\bar{n}$ to maximize her overall profit.

In order to solve for the equilibrium, we first derive consumers' choices of buying quantity q and ad viewing quantity n, which are summarized in Lemma 6. To maintain continuity with the main model, we still classify consumer choice cases into CS, Hybrid, and RA, but different from the main model, these cases indicate consumers' preferences instead of their actual choices. For example, RA here means consumers all prefer reward ads to content purchasing (though they may still buy premium content when reaching $\bar{n}$). We further denote subcases using the "type-choice" format, where type $\in \{L, H, Both\}$ denotes the type of consumers' nuisance costs, and choice $\in \{buy, mix, max, interior, none\}$ denotes the highest-valuation customer's choice, which could be buying premium content (*buy*), viewing maximum number of ads and buying premium content (*mix*), viewing the maximum number of ads but not buying premium content (*max*), viewing less than the maximum number of ads (*interior*), and neither viewing nor buying (*none*).

LEMMA 6. (CONSUMERS' CHOICES IN THE EXTENDED MODEL). *Depending on the app developer's choices of p, r, and $\bar{n}$, there are the following cases for consumers' choices*[7]*:*

- *Case CS when $p < \frac{c_L}{r}$: both c_L and c_H consumers prefer buying premium content.*
- *Case Hybrid when $\frac{c_L}{r} < p < \frac{c_H}{r}$: c_L consumers prefer viewing ads and c_H consumers prefer buying*

premium content
- ○ *Case Hybrid 1 (L-mix, H-buy) when $p + r\bar{n} < V$: high-valuation c_L consumers view the maximum amount of reward ads and buy some premium content; high-valuation c_H consumers buy premium content.*
- ○ *Case Hybrid 2 (L-max, H-buy) when $p + r\bar{n} > V$ and $\frac{c_L}{r} + r\bar{n} < V$: high-valuation c_L consumers view the maximum amount of reward ads but do not buy premium content; high-valuation c_H consumers buy premium content.*
- ○ *Case Hybrid 3 (L-interior, H-buy) when $\frac{c_L}{r} + r\bar{n} > V$: high-valuation c_L consumers view less than maximum amount of reward ads and do not buy premium content; high-valuation c_H consumers buy premium content.*

- *Case RA when $p > \frac{c_H}{r}$: both c_L and c_H consumers prefer viewing ads*
  - ○ *Case RA 1 (Both-mix) when $p + r\bar{n} < V$: both high-valuation c_L consumers and high-valuation c_H consumers view the maximum amount of reward ads and buy some premium content.*
  - ○ *Case RA 2 (Both-max) when $p + r\bar{n} > V$ and $\frac{c_H}{r} + r\bar{n} < V$: both high-valuation c_L consumers and high-valuation c_H consumers view the maximum amount of reward ads but do not buy premium content.*
  - ○ *Case RA 3 (L-max, H-interior) when $\frac{c_H}{r} + r\bar{n} > V$, $\frac{c_L}{r} + r\bar{n} < V$, and $\frac{c_H}{r} < V$: high-valuation c_L consumers view the maximum amount of reward ads but do not buy premium content; high-valuation c_H consumers view less than maximum amount of reward ads and do not buy premium content.*
  - ○ *Case RA 4 (Both-interior) when $\frac{c_L}{r} + r\bar{n} > V$ and $\frac{c_H}{r} < V$: both high-valuation c_L consumers and high-valuation c_H consumers view less than maximum amount of reward ads and do not buy premium content.*
  - ○ *Case RA 5 (L-max, H-none) when $\frac{c_L}{r} + r\bar{n} < V$ and $\frac{c_H}{r} > V$: high-valuation c_L consumers view the maximum amount of reward ads but do not buy premium content; c_H consumers neither view rewards ads nor buy premium content.*
  - ○ *Case RA 6 (L-interior, H-none) when $\frac{c_L}{r} + r\bar{n} > V$ and $\frac{c_H}{r} > V$: high-valuation c_L consumers view less than maximum amount of reward ads and do not buy premium content; c_H consumers neither view rewards ads nor buy premium content.*

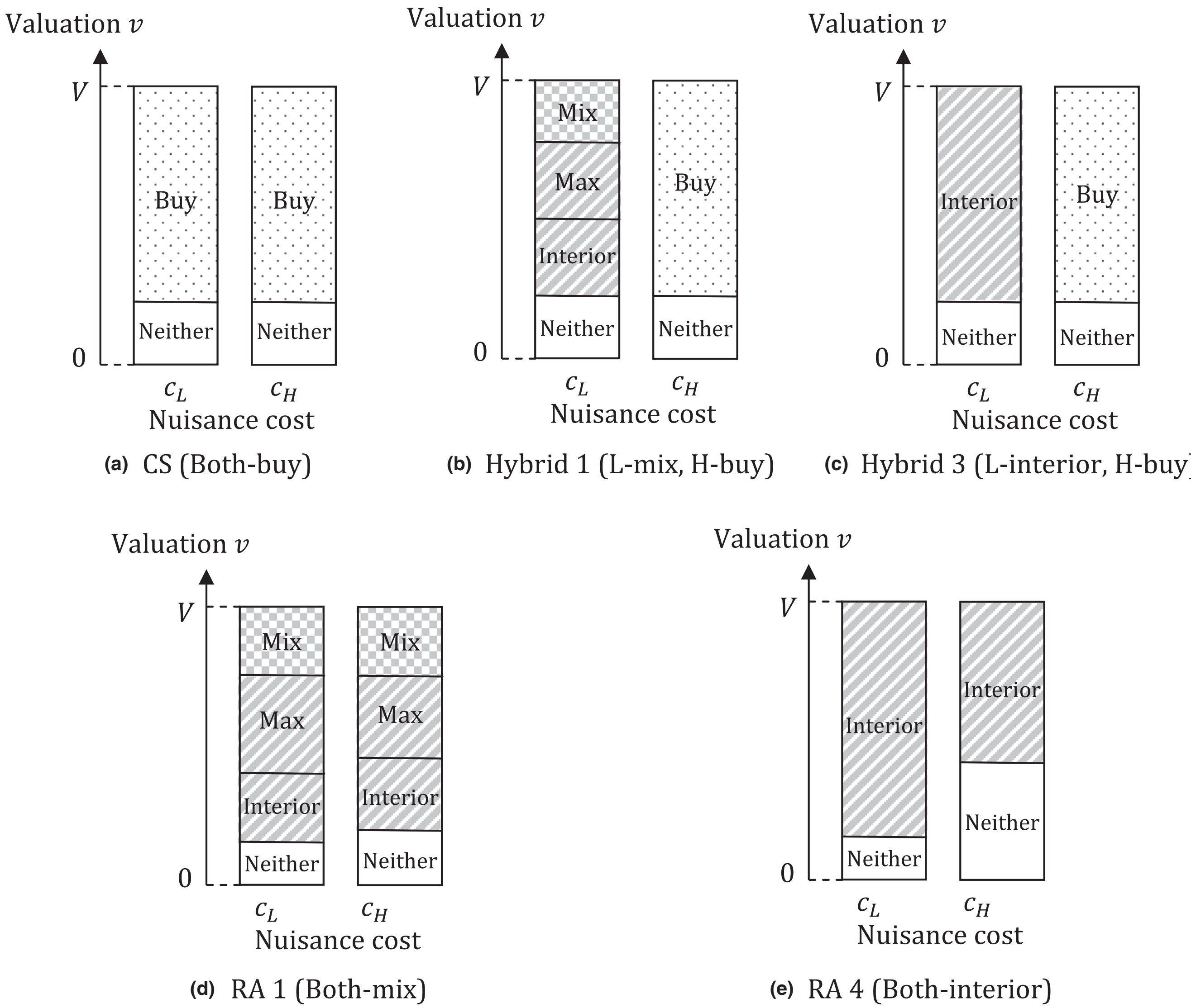
Lemma 6 reveals the similarities and differences in consumers' choices between the main and the extended models. Similar to the main model, no one prefers viewing ads, when unit price for premium content p is low (Case CS); c_L consumers prefer viewing ads and c_H consumers prefer buying, when p is medium (Hybrid cases); no one prefers buying when p is high (RA cases). Different from the main model, when the app developer sets maximum ad viewing quantity $\bar{n}$ in the extended model, consumers' choices of buying quantity q and ad viewing quantity n become more nuanced. For example, in Case Hybrid 1 (L-mix, H-buy), high-valuation c_L consumers with $v \in (p + r\bar{n}, V)$ would like to view more reward ads than $\bar{n}$, but they cannot do so due to the ad viewing limit set by the app developer. As a result, they choose viewing quantity of $n = \bar{n}$ and then buy additional $q = v - p - r\bar{n}$ of premium content. Note that some consumers may choose both buying premium content and viewing reward ads in the extended model, which does not occur in the main model.

Next, we analyze the app developer's optimal strategies. Since the app developer's profit maximization problem is analytically intractable, we use a numerical approach. To ensure representativeness of the numerical solutions, we conduct extensive numerical analyses for a wide range of parameter values.[8] Overall, we solve the app developer's profit maximization problem for 348,840 sets of parameter values. We summarize the app developer's optimal strategies in Numerical Result 1.

Numerical Result 1. (App Developer's Optimal Strategies in the Extended Model). *When the app developer chooses p, r, and $\bar{n}$, there are five possible equilibrium cases: CS (Both-buy), Hybrid 1 (L-mix, H-buy), Hybrid 3 (L-interior, H-buy), RA 1 (Both-mix), and RA 4 (Both-interior).*

Figure 5 illustrates consumers' decisions in all the possible cases of the app developer's optimal strategies. Cases CS (Both-buy), Hybrid 3 (L-interior, H-buy), and RA 4 (Both-interior) correspond to Cases CS, Hybrid, and RA in the main model, respectively. Interestingly, we find two new equilibria: Hybrid 1 (L-mix, H-buy) and RA 1 (Both-mix). In these two equilibria, the app developer sets a binding maximum ad viewing quantity ($\bar{n}$), which results in two new consumer choice types: some "Mix" choice types in which the highest-valuation consumers not only view the maximum number of ads but also buy premium content ("Mix" in Figure 5b and d), and some "Max" choice cases, in which high-valuation consumers view the maximum number of ads but do not buy premium content ("Max" in Figure 5b and d). Unlike the main model, consumers in the "Mix" case both view reward ads and buy premium content.

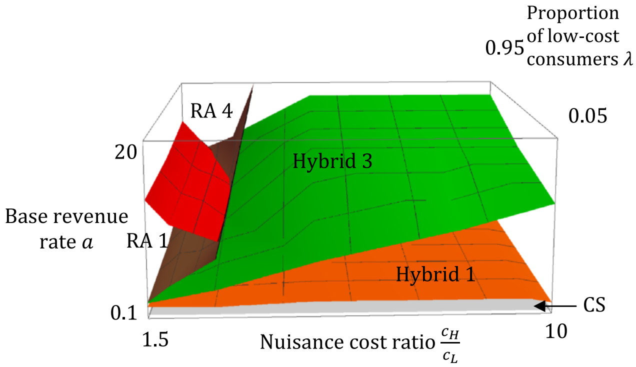
Figure 5 Consumer Decisions under the App Developer's Optimal Strategies in the Extended Model



***Notes*: Mix = Mixture of viewing maximum number of ads and buying premium content, Max = View the maximum number of ads but not buy premium content, Interior = View less than the maximum number of ads, Buy = Buy premium content, and Neither = Neither view reward ads nor buy premium content.**

Figure 6 illustrates the market conditions for the five possible cases of the app developer's optimal strategies. Similar to the main model, the app developer should provide reward advertising when the advertising channel is sufficiently efficient, that is, base ad revenue rate a is higher than a threshold (this threshold takes a different form compared to the one in the main model). When the app developer does provide reward advertising, a pure reward-ads strategy (charging a high unit price p such that all consumers prefer viewing reward ads over buying premium content) is optimal if base ad revenue rate a is relatively high and nuisance cost ratio $\frac{c_H}{c_L}$ is relatively low (RA 1 and RA4). Otherwise, a hybrid strategy (charging a medium unit price p such that low-cost consumers prefer viewing reward ads and high-cost consumers prefer buying premium content) is optimal. Different from the main model, as illustrated in Figure 6, two new equilibria (Hybrid 1 and RA 1) emerge where the reward ads limit is binding.

Figure 6 App Developer's Optimal Strategies in the Extended Model [Color figure can be viewed at wileyonlinelibrary.com]



***Note*: Here we set $V = 10$ and $b = 1$. Other parameter values result in qualitatively the same figures.**

Next, we numerically explore when the app developer should set a maximum ad viewing quantity and how the optimal maximum ad viewing quantity change with a few underlying model parameters. The results are summarized in Numerical Result 2.

NUMERICAL RESULT 2. (PROPERTIES OF THE OPTIMAL MAXIMUM AD VIEWING QUANTITY). *The app developer's optimal maximum ad viewing quantity $\bar{n}^*$ has the following properties:*

(a) is nonbinding (i.e., the app developer sets a high enough $\bar{n}^*$ such that no one is prevented from viewing another ad), when base ad revenue rate a is higher than a threshold; and binding, otherwise. Furthermore, when $\bar{n}^*$ is binding, $\bar{n}^*$ increases in a.
(b) is nonbinding when diminishing speed b is lower than a threshold; and binding, otherwise. Furthermore, when $\bar{n}^*$ is binding, $\bar{n}^*$ decreases in b.
(c) is nonbinding when proportion of low-cost consumers λ is in the medium range; and binding, otherwise.
(d) is nonbinding when nuisance cost ratio $\frac{c_H}{c_L}$ is in the medium range; and binding, otherwise.

When base ad revenue rate a is high enough or diminishing speed b is low enough, the optimum maximum ad viewing quantity is nonbinding (indicated by open circles for Case Hybrid 3 in Figure 7a and b). In other words, the app developer effectively does not set any limit on ad viewing in these cases. The latter confirms our earlier claim that the app developer would not limit ad views if the ad revenue rate is constant. Furthermore, as the advertising channel becomes more efficient, that is, as base ad revenue rate a increases (Figure 7a) or diminishing speed b decreases (Figure 7b), the app developer should set a higher $\bar{n}$ to allow consumers view more reward ads. This result is intuitive since in such cases, it is profitable to increase ad views by raising $\bar{n}$.

Numerical Results 2c and 2d explore the impacts of two consumer heterogeneity characteristics (proportion of low-cost consumers λ and nuisance cost ratio $\frac{c_H}{c_L}$) on the optimal $\bar{n}^*$. Similar to Case Hybrid 3 (Figure 7a and b), in Case RA 4 (Figure 7c and d), the app developer does not impose a reward-ads limit (or setting a nonbinding $\bar{n}^*$, as indicated by open circles). As illustrated in Figure 7c, when there is a small proportion of low-cost consumers (a low λ) or a large proportion of low-cost consumers (a high λ), consumers are highly homogeneous in terms of their nuisance costs and consequently they would like to view similar amount of reward ads. As a result, it is effective for the app developer to impose a binding limit $\bar{n}$ to avoid excessive reward-ads viewing. In contrast, consumers are highly heterogeneous in nuisance costs when λ is in the medium range. As a result, setting a uniform $\bar{n}$ for both low- and high-cost consumers is no longer effective because they would like to view different amount of reward ads. Therefore, the app developer ends up not imposing a limit (Case RA 4).

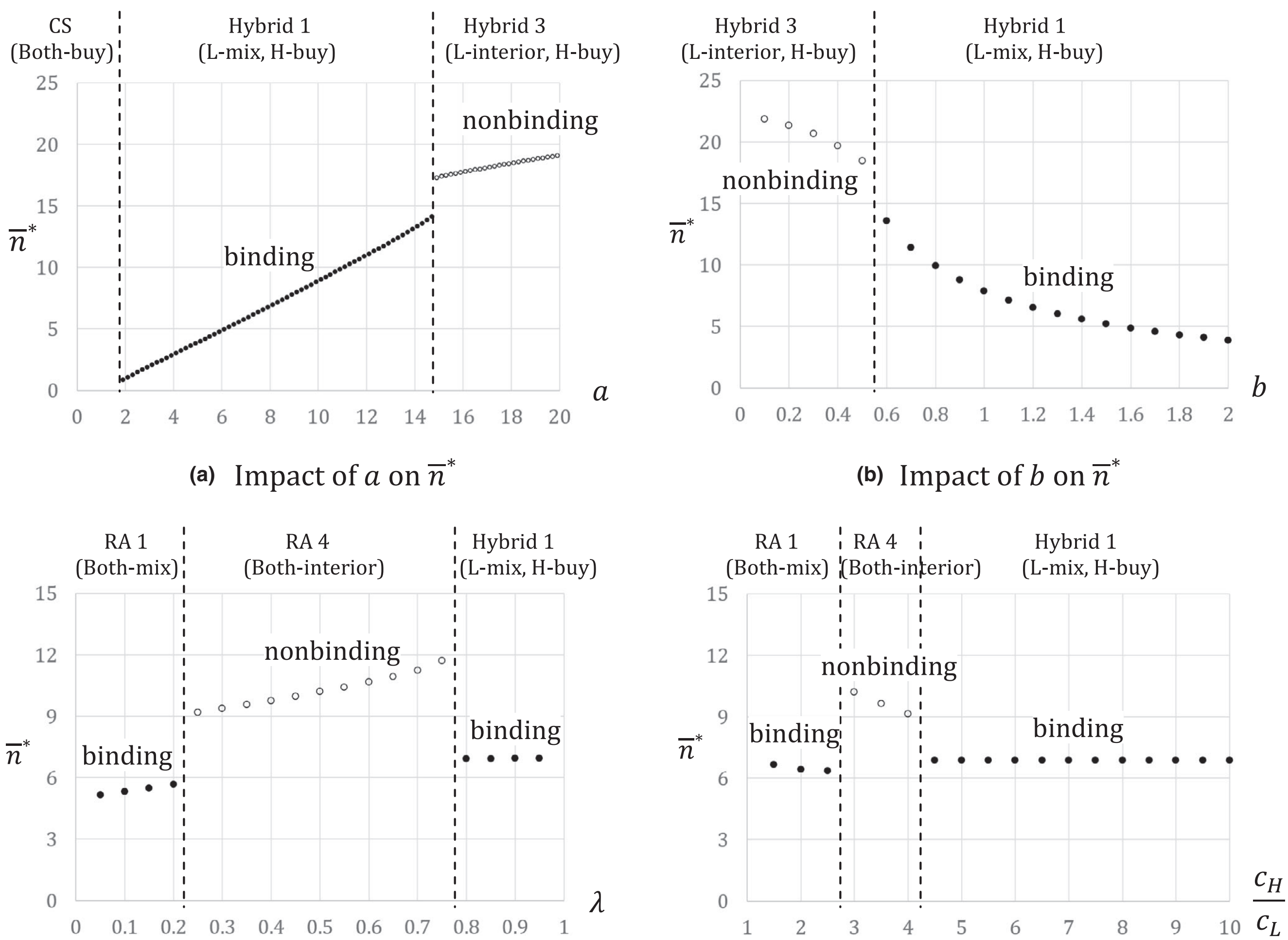
As illustrated in Figure 7d, when both low- and high-cost consumers have relatively low nuisance costs ($\frac{c_H}{c_L}$ is low), they both tend to view many ads. Setting a uniform $\bar{n}$ is beneficial for preventing excessive reward-ads viewing. When consumers become more heterogeneous in their nuisance costs, but not enough to make the hybrid strategy optimal, setting a uniform $\bar{n}$ for both low- and high-cost consumers is no longer effective. As a result, the app developer does not limit ad viewing quantity. When consumers become highly heterogeneous in their nuisance costs, the app developer is better off adopting the hybrid strategy, that is, inducing high-cost consumers to buy but low-cost ones to view ads. In such a case, it is optimal for the app developer to impose a limit optimized for low-cost consumers.

8. Discussion and Conclusion

Motivated by the emergence of reward advertising for app developers, we model reward advertising along with premium content selling, and study consumer and app developer decisions. As the first economic modeling and analysis of reward advertising, this research answers the questions such as (i) when an app developer should offer reward advertising, (ii) if an app developer does offer reward advertising, should she offer it alone or combine it with premium content selling? (iii) how to choose the optimal reward rate, and (iv) when to set a limit on maximum number of reward ads.

Our economic analyses of reward ads generate several insights of academic and practical

Figure 7 Properties of Optimal Maximum Ad Viewing Quantity $\bar{n}^*$



(a) Impact of a on $\bar{n}^*$

(b) Impact of b on $\bar{n}^*$

(c) Impact of λ on $\bar{n}^*$

(d) Impact of $\frac{c_H}{c_L}$ on $\bar{n}^*$

***Notes*: (a) $V = 10$, $b = 1$, $\lambda = 0.5$, and $\frac{c_H}{c_L} = 5$. $\bar{n}^*$ is undefined for the CS case. (b) $V = 10$, $a = 9$, $\lambda = 0.5$, and $\frac{c_H}{c_L} = 5$. (c) $V = 10$, $b = 1$, $a = 8$, and $\frac{c_H}{c_L} = 3$. (d) $V = 10$, $b = 1$, $a = 8$, $\lambda = 0.5$, $c_L = 1$, and vary $\frac{c_H}{c_L}$ by changing c_H.**

importance. First, our results highlight the core mechanism of reward advertising: creating and leveraging "efficient" advertisements: reward advertising is profitable as long as the reward ad revenue exceeds some consumer's nuisance cost. Reward advertising has a few advantages in achieving efficiency: (i) consumers with low nuisance cost will view reward ads, and in doing so, they can choose ads they would enjoy the most; (ii) because of the explicit value exchange, consumers are more willing to engage with the advertisement, thus yielding better outcomes for advertisers. The efficiency principle suggests that simply bribing consumers into viewing reward ads may not work: reward ads must be carefully engineered to ensure low nuisance cost and high engagement with the ads.

Second, our findings suggest that reward advertising can be a good addition to the "freemium" strategy. In what we call a hybrid strategy, reward advertising converts paying customers who have low nuisance costs into reward ads viewers, while keeping other paying customers the same. By setting an appropriate reward rate, an app developer can keep consumers as happy and extract the additional surplus generated from an efficient reward ad. Our results show that such a hybrid strategy should be used when the ad revenue rate is not too high and consumers are highly heterogeneous in nuisance costs—which we believe describes the majority of real-world settings. Moreover, the core mechanism of reward does not limit the type of content that can be used as a reward; we thus infer that the hybrid strategy could be used for a large variety of different apps.

We also reveal an interesting trade-off when choosing the reward rate: a high reward rate has both a positive "stronger incentive" effect and a negative "accelerated satiation" effect—the latter arises naturally because the value of each additional unit of premium content diminishes as consumers obtain more of it. This trade-off is most evident when the app developer pursues a pure reward-ad strategy, which is only optimal when it is more beneficial to incentivize both low- and high-cost consumers to choose reward ads (provided that they have high enough valuation for premium content). In such a scenario, high-cost consumers need high reward rates to overcome their nuisance costs. Low-cost ones, on the other hand, may be best motivated by lower reward rates because of accelerated satiation. We show that when the ratio of low-cost consumers increases, the optimal reward rate should be lower. When the ratio of low-cost consumers is high enough, the optimal reward rate is so low such that total consumer surplus is lower than a pure content-selling or a hybrid strategy. Conversely, when the ratio of low-cost consumers is low enough, a pure reward-ad strategy can also produce higher consumer surplus. These findings thus provide useful guidelines and cautionary tales for app developers on how to set the optimal reward rate.

In practice, consumers' nuisance costs can be proxied by consumer demographics such as (estimated) income, education level, occupation, etc. App developers can use such customer intelligence to get an idea about the distribution of nuisance cost among the app users, which can then be used to make a decision on revenue models. Additionally, in some cases, consumer nuisance costs can also be estimated from the ad viewing behavior itself, e.g., by estimating consumer elasticities in ad viewing.

One can think of cases where consumers' valuation of the premium content is positively correlated with their ad nuisance cost. We point out, however, even with such a positive correlation, the main driving forces in our model remain the same and, thus, our main results should remain qualitative the same. Each consumer will still rely on the same criterion—whichever leads to a lower cost of acquiring one unit of the premium content—to decide whether she will choose direct purchase or viewing reward ads. Meanwhile, the conditions required for the three strategies—pure content-selling, hybrid, and pure reward advertising—also remain the same. This invariance in boundary conditions suggests that the developer still faces similar trade-offs when choosing the price of the premium content and the reward rate. Therefore, our paper's conclusions will not change qualitatively even when the valuation is positively correlated with ad nuisance cost.

We find that a limit on the number of reward ads per consumer can be used to combat "excessive" viewing of reward ads, that is, when the ad revenue rate declines such that it is no longer profitable to offer the consumer another reward ad. Such a limit is only useful when ad revenue rate declines fast enough, and the base ad revenue rate is neither too high (otherwise consumers will not reach the point of "excess" from the developer's perspective) nor too low (otherwise reward advertising is not optimal). Furthermore, a reward ads limit is more effective when consumers who choose to view reward ads are more homogenous in nuisance costs, e.g., in the hybrid case where only low-cost consumers view reward ads, or in the pure reward-ads cases where high-cost customers dominate or are not too different from low-cost customers in nuisance costs. Thus, a reward-ads limit is more likely to be optimal in the hybrid cases than in the pure reward-ads cases.

As a first economic analysis of reward advertising, we have made a few simplifying assumptions. We have abstracted way the heterogeneity of premium content to focus on consumers' choice between buying premium content and viewing reward ads in exchange for premium content. That said, our model does accommodate a variety to different apps, which may be valued differently by their customers, have a different distribution of nuisance costs, or have different potential in generating ad revenue. We also limit the alternative monetization mechanism to direct selling of premium content. Though the freemium model is popular among mobile apps, we acknowledge that there are other monetization mechanisms (perhaps in a broad context than mobile apps), such as a subscription model. How reward advertising interacts with other monetization mechanisms (e.g., subscription model) is for future research. Finally, we focus on the role of reward advertising as a content monetization strategy for mobile app developers. Reward advertising can also be used in broader contexts (e.g., as part of the customer loyalty program) that can be studied in future work.

To summarize, this research contributes to the academic literature by modeling and analyzing a novel monetization mechanism for mobile app developers—reward advertising. Our results provide practical guidance for app developers on when to use reward advertising, how to choose optimal rewards rates and set reward limits, and what to expect about its impact on consumers. We also contribute to the broader literature of monetization mechanisms by showing that one can gainfully leverage consumer heterogeneity in nuisance cost.

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Notes

[1]The other main source of revenue for free apps is in-app purchases.
[2]We thank the anonymous reviewers for suggesting us to look into this interesting phenomenon.
[3]We use premium content to broadly refer to premium content, service, and other value-added features.
[4]We note that in practice, app developers also provide some content for free. We consider that the amount of content offered for free is fixed at its optimal point and does not interact with consumers' choice between buying premium content and viewing reward ads in exchange for additional premium content.
[5]We note that the assumption of diminishing marginal returns can be further relaxed. Our results do not rely so much on the marginal utility decreasing monotonically as on the marginal utility *eventually decreasing*. For example, our main results are qualitatively similar if the marginal utility first increases then decreases, which mimic the case where consumers may develop increased appreciation for the premium content initially.
[6]The numbers of premium consumers, the premium content consumptions, and the consumer surpluses may not bse the same under the pure content-selling strategy and the hybrid one when nuisance cost distribution changes. However, the main results regarding the app developer's optimal strategies remain qualitatively the same.
[7]Specific consumer segments and their corresponding choices of buying quantity q and ad viewing quantity n can be found in the proof of Lemma 6 in online Appendix S1.
[8]We numerically solve the app developer's profit maximization problem for the following parameter values. We vary base ad revenue rate a from 0.1 to 1.5 with increments of 0.1 and then from 2 to 20 with increments of 1, resulting in 34 steps for a. We vary diminishing speed b from 0.1 to 3 with increments of 0.1, resulting in 30 steps for b. We vary proportion of low-cost consumers λ from 0.05 to 0.95 with increments of 0.05, resulting in 19 steps for λ. To explore different values of nuisance cost ratio c_H/c_L, we set $c_L = 1$ and vary c_H/c_L by changing c_H from 1.5 to 10 with increments of 0.5, resulting in 18 steps for c_H.

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Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Appendix S1. Proofs.